

Fiscal and financial system in  
japan

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06/02/2009

**THE TERM STRUCTURE OF  
INTEREST RATES (1)  
(MISHKIN, CHAPTER 6)**

## Term Structure of Interest Rates

Interest rates on government bonds  
(As of May 11, 2009)

Maturity date (years to maturity)	Interest rate
May 2011 (2)	0.4%
March 2014 (5)	0.9%
March 2019 (10)	1.3%
March 2029 (20)	2.1%
March 2039 (30)	2.3%

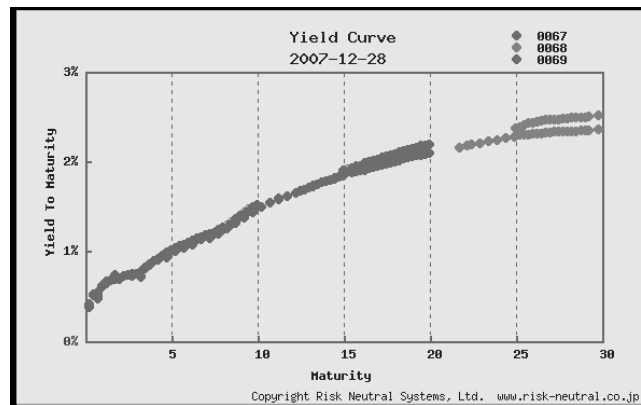
Source: Nikkei Shinbun, May 11, 2009

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## Yield Curve

A plot of the yields on bonds with different maturities,  
but the same risk and liquidity

An example of yield curve for government bonds

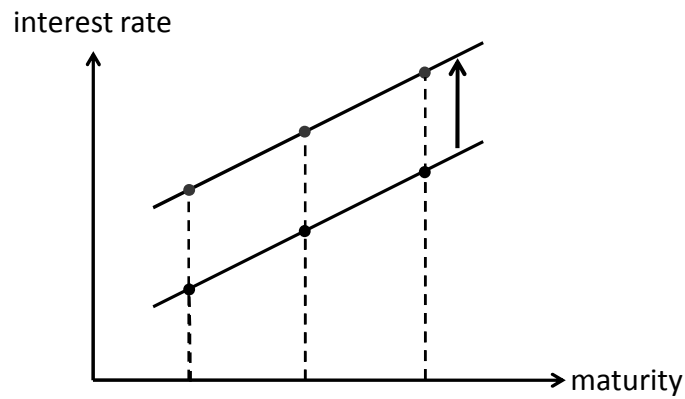


Source: Risk Neutral Systems, Ltd.

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## Three Empirical Facts

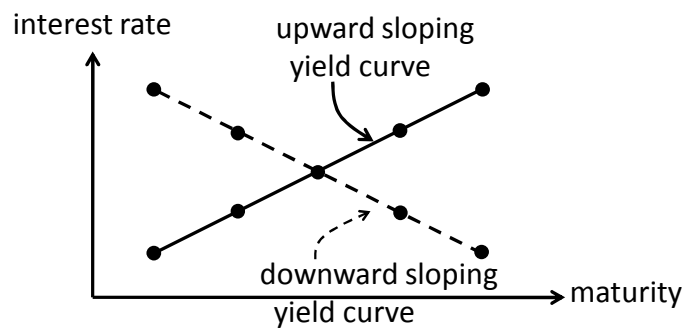
**Fact 1** Interest rates on bonds of different maturities move together over time.



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**Fact 2** When short-term interest rates are low, yield curves tend to slope upward; When short-term interest rates are high, yield curves tend to slope downward.

**Fact 3** Yield curves almost always slope upward.



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## Term Structure and Economic Theory

Any theory that explains all of these empirical facts CONSISTENTLY?

▶ Three theories advanced by economists

- 1 Expectations Theory**
- 2 Segmented Markets Theory**
- 3 Liquidity Premium Theory**

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## Assumptions of the Expectations Theory

### A.1 Perfect Substitutes

For investors, bonds with different maturities are “perfect substitutes”.

Investors do not prefer bonds with one maturity to another.

### A.2 Risk-Neutrality

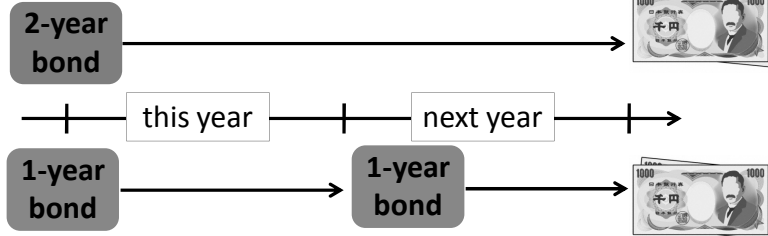
Investors are “risk-neutral.”

Investors are interested only in the expected return, neglecting the variability or uncertainty of the future short-term interest rates.

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## Perfect Substitutes

### Buy-and-Hold Strategy

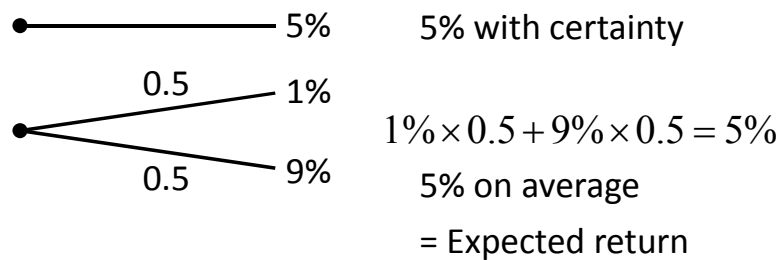


### Roll-Over Strategy

For investors, two strategies are equivalent, and thus perfectly substitutable.

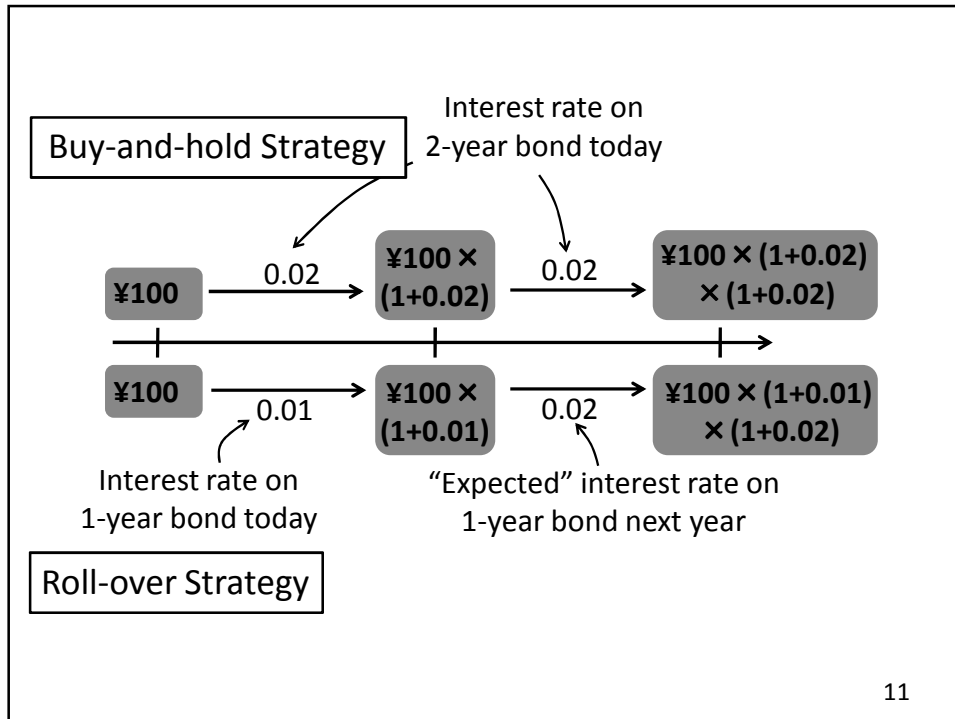
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## Risk-Neutrality



For investors, two investments are equivalent.

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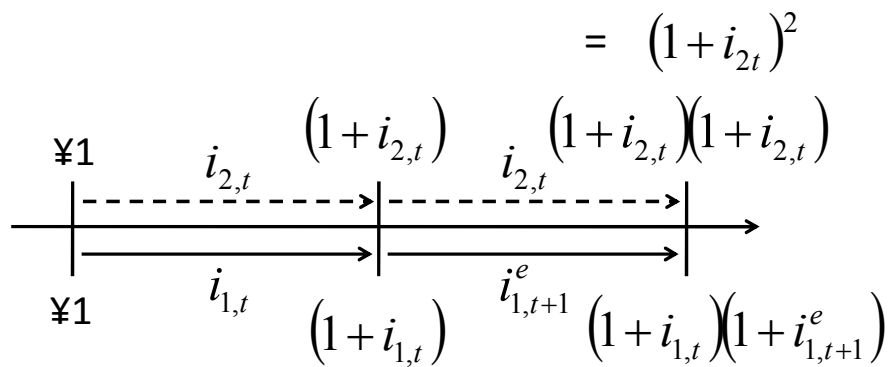
Under the assumption of perfect substitutability and risk-neutrality ...

Yield from roll-over < Yield from buy-and-hold

- ▶ Everyone substitutes 2-year bonds for 1-year bonds.
- ▶ Price of 2-year bond ↑      Price of 1-year bond ↓  
     Interest rate on 2-year bond ↓      Interest rate on 1-year bond ↑

The yields from the two strategies must be equal, or the relationship among the three interest rates must ensure the equality.      Term structure of interest rates

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$i_{2,t} \dots$  today's interest rate on an 2-year bond

$i_{1,t} \dots$  today's interest rate on an 1-year bond

$i_{1,t+1}^e \dots$  interest rate on a one-year bond next year that we expect "today"

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$$(1 + i_{1,t})(1 + i_{1,t+1}^e) = (1 + i_{2,t})^2$$

$$1 + i_{1,t} + i_{1,t+1}^e + i_{1,t}i_{1,t+1}^e = 1 + 2i_{2,t} + (i_{2,t})^2$$

$$i_{1,t} + i_{1,t+1}^e = 2i_{2,t}$$

$$\frac{i_{1,t} + i_{1,t+1}^e}{2} = i_{2,t}$$

The two-period rate must equal the average of the two one-period rates.

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To be more general ...

$$\frac{i_{1,t}^e + i_{1,t+1}^e + i_{1,t+2}^e + \dots + i_{1,t+(n-1)}^e}{n} = i_{n,t}$$

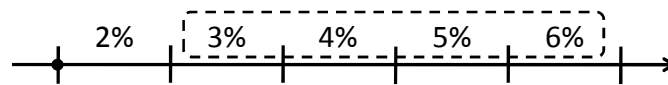
$n$ -year bond rate must equal  
the average of  $n$  one-year bond rates.

$i_{1,t+i}^e \dots$  interest rate on a one-year bond  
 $i$  year ahead that we expect "today"

$i_{n,t}$  ... today's interest rate on an  $n$ -year bond

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Numerical Example(1): Expect interest rates to rise



Today's interest rate on a 2-year bond is ...

$$(2\% + 3\%)/2 = 2.5\%$$

For a 3-year bond

$$(2\% + 3\% + 4\%)/3 = 3\%$$

For a 4-year bond

$$(2\% + 3\% + 4\% + 5\%)/4 = 3.5\%$$

For a 5-year bond

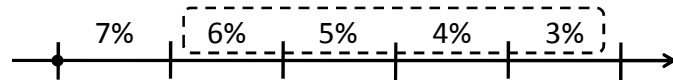
$$(2\% + 3\% + 4\% + 5\% + 6\%)/5 = 4\%$$

**Upward sloping  
yield curve**

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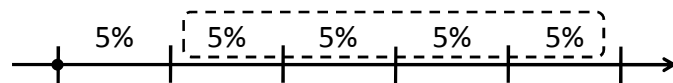
Numerical Example(2): Expect interest rates to fall



$$\left. \begin{aligned} (7\% + 6\%)/2 &= 6.5\% \\ (7\% + 6\% + 5\%)/3 &= 6\% \\ (7\% + 6\% + 5\% + 4\%)/4 &= 5.5\% \\ (7\% + 6\% + 5\% + 4\% + 3\%)/5 &= 5\% \end{aligned} \right\} \text{Downward sloping yield curve}$$

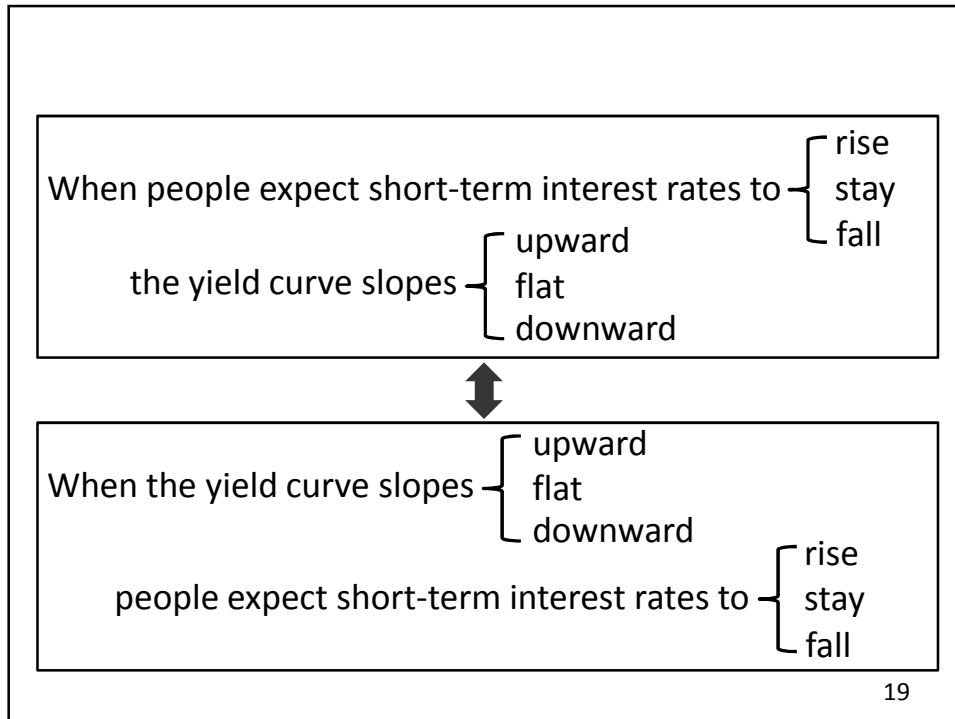
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Numerical Example(3): Expect interest rates to stay



$$\left. \begin{aligned} (5\% + 5\%)/2 &= 5\% \\ (5\% + 5\% + 5\%)/3 &= 5\% \\ (5\% + 5\% + 5\% + 5\%)/4 &= 5\% \\ (5\% + 5\% + 5\% + 5\% + 5\%)/5 &= 5\% \end{aligned} \right\} \text{Flat yield curve}$$

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## Expectations Theory and “Fact 2”

Short-term rate is low today

It is more likely to rise (back to some normal level)

People expect short-term rates to rise

**Upward sloping** yield curve occurs

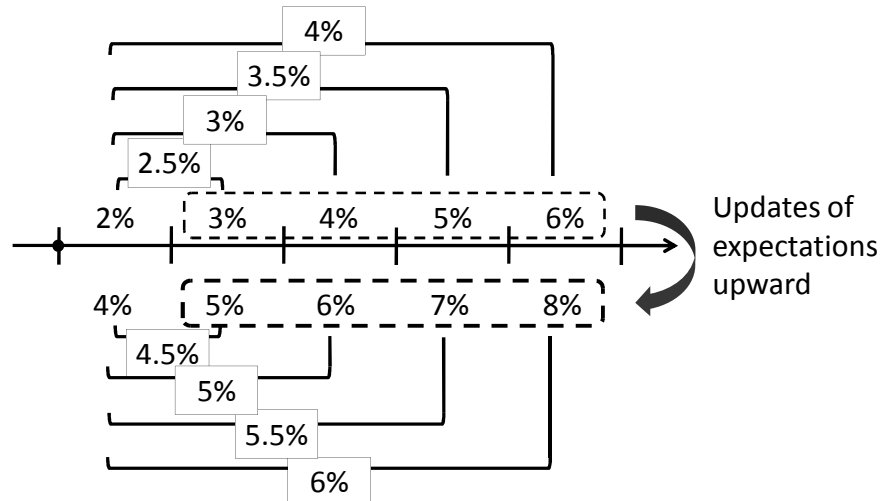
Short-term rate is high today

It is more likely to fall (back to some normal level)

People expect short-term rates to fall

**Downward sloping** yield curve occurs

## Expectations Theory and “Fact 1”



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## Mid-Term Exam

- You will be mainly asked to explain how relationships among economic variables are derived.
  - Ex. Derive the relationship between the price of a bond and its interest rate.
- You will also be asked to explain some important concepts in monetary economics.
- You can be asked to perform some simple calculation.
- I will not be strict in evaluation this time. My comments on your answers will help you to do better in the final exam.

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