3543 Fiscal and Financial System in Japan A / KC3002 International Finance

Fall 2013

Lecture 5(Oct 18)
Interest Rates (cont.)

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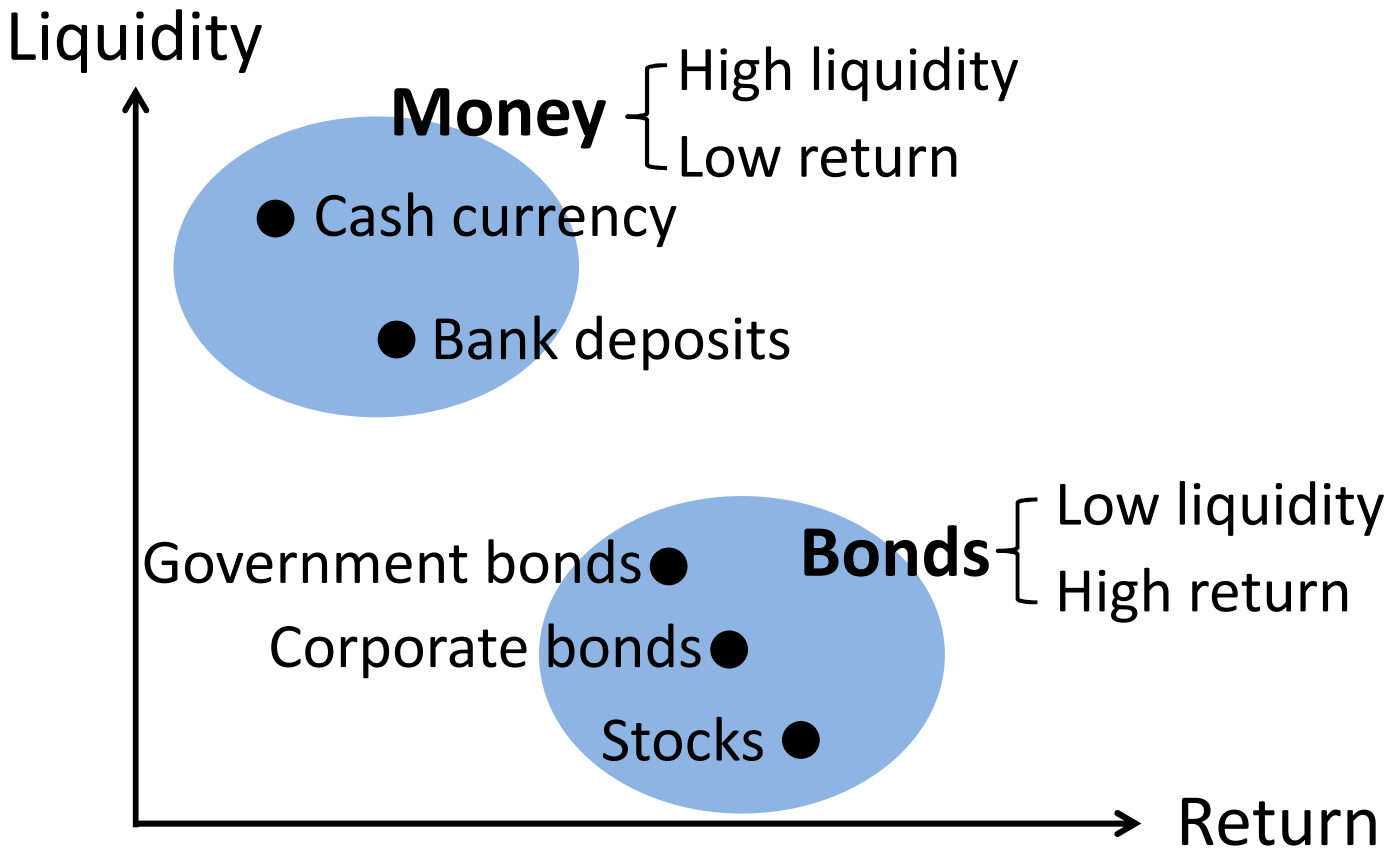
Rev 1

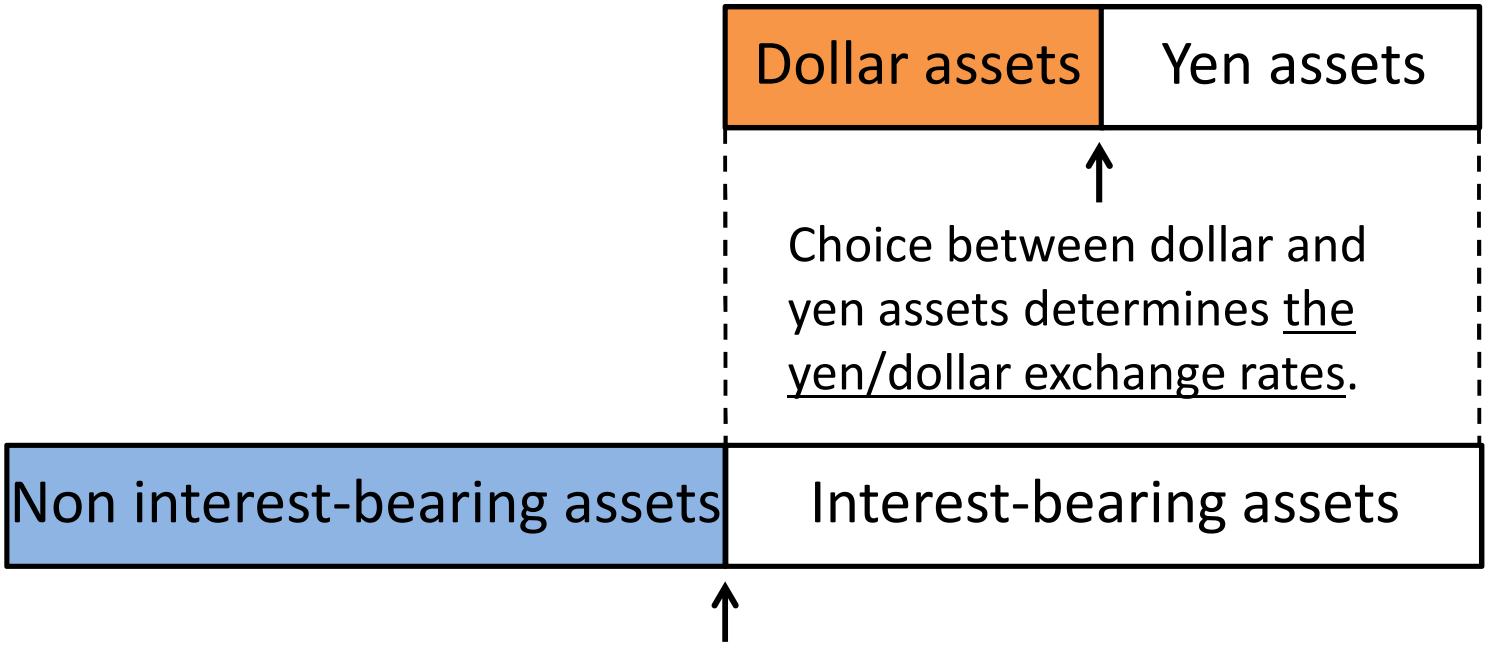
Money and Bonds



Liquidity： the ease and speed with which an asset can be used to purchase goods/services

Rev 2

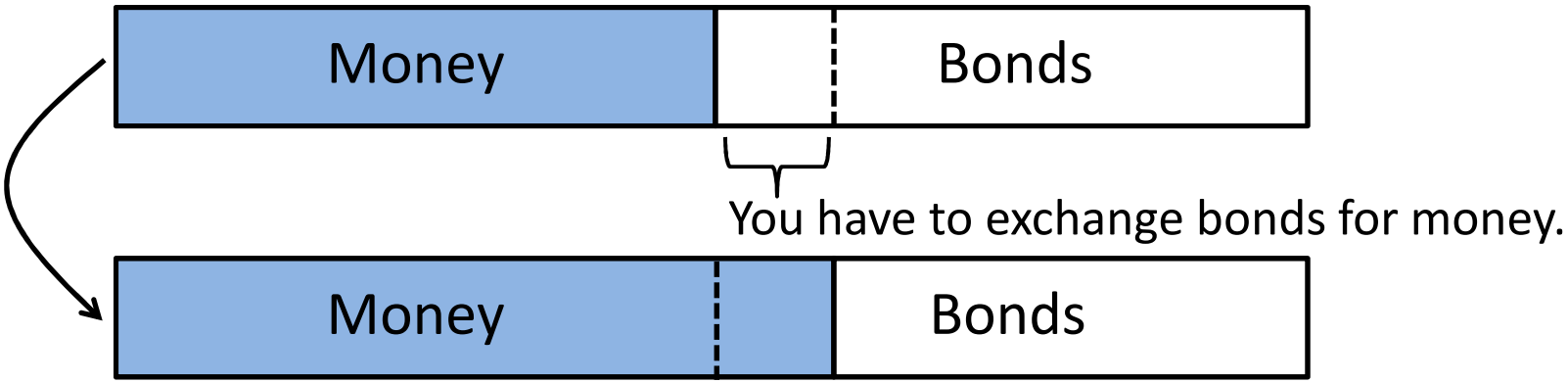
Portfolio Decision



Rev 3

Money Demand and Interest Rates

In order to hold more money …



Holding more money ⟷ Reducing bonds

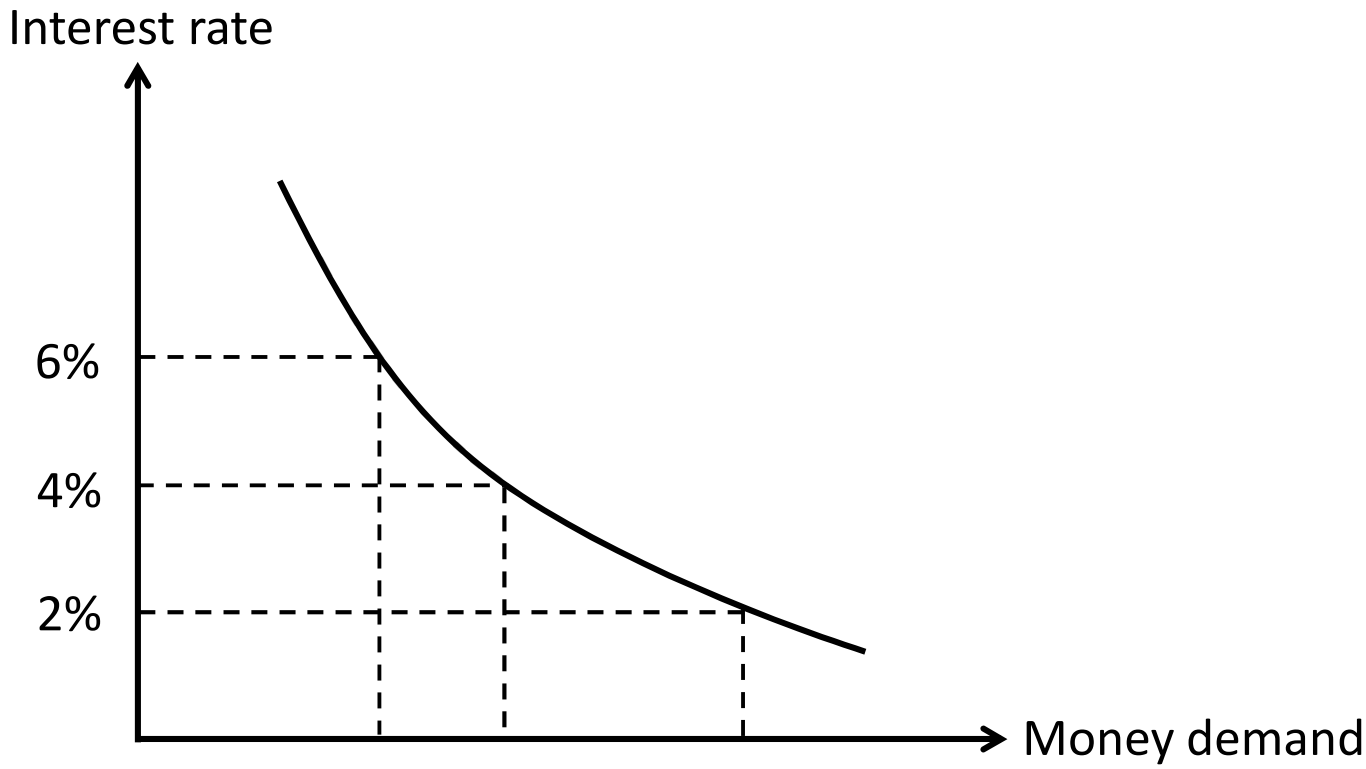
⟷ Giving up interests on those bonds

The higher the interest rates,
the smaller the demand for money.

Rev 4

Demand for Money

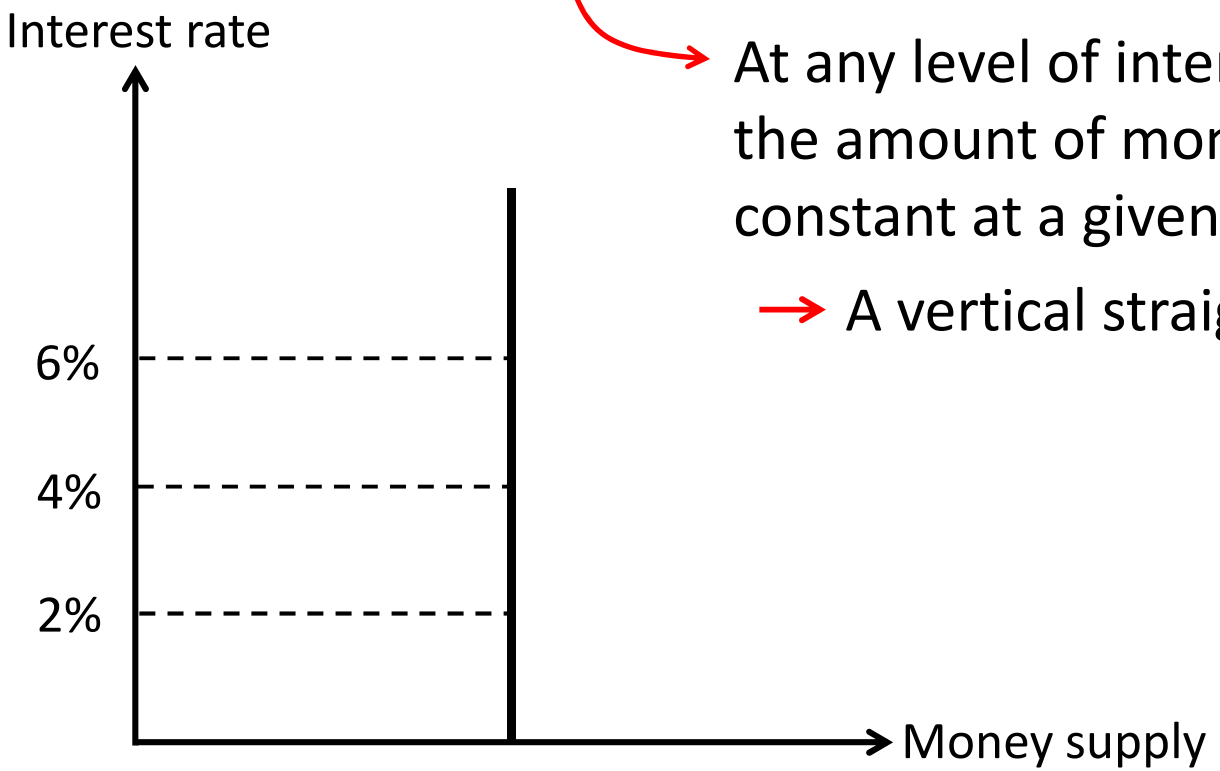
Demand for money is negatively related with interest rates on bonds.



Rev 5

Supply of Money

Supply of money is controlled by the central bank.
The central bank bases its decision on policy consideration, thus is independent of interest rates.

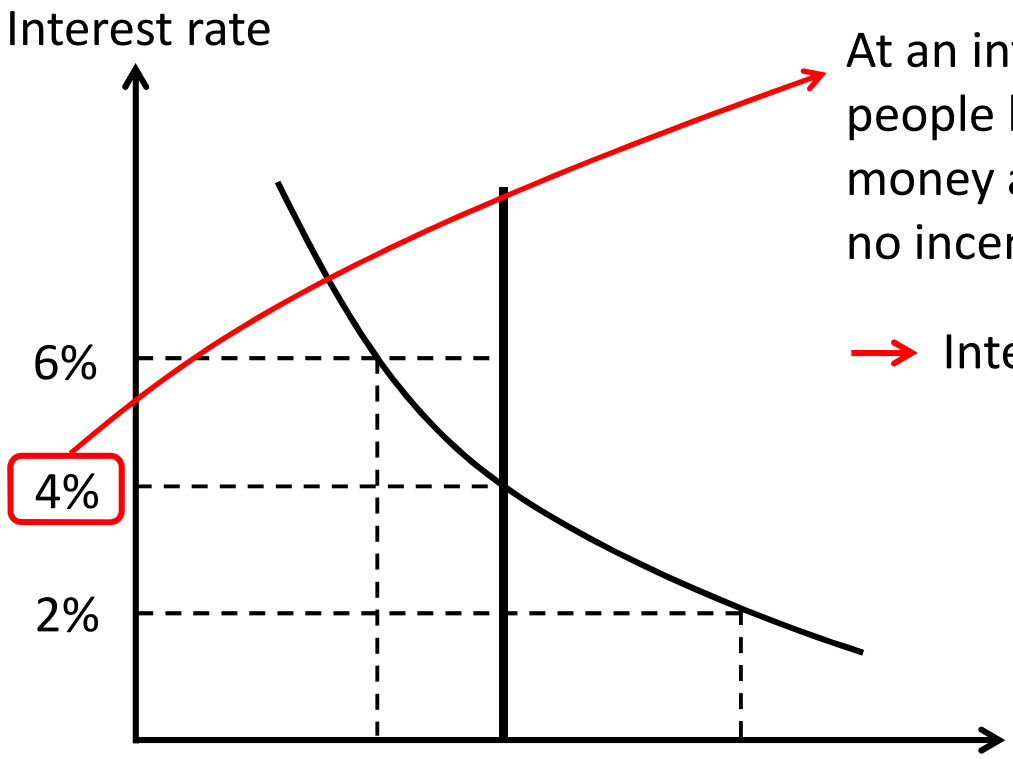


At any level of interest rate, the amount of money supplied is constant at a given level.

→ A vertical straight line.

Rev 6

Equilibrium Interest Rates

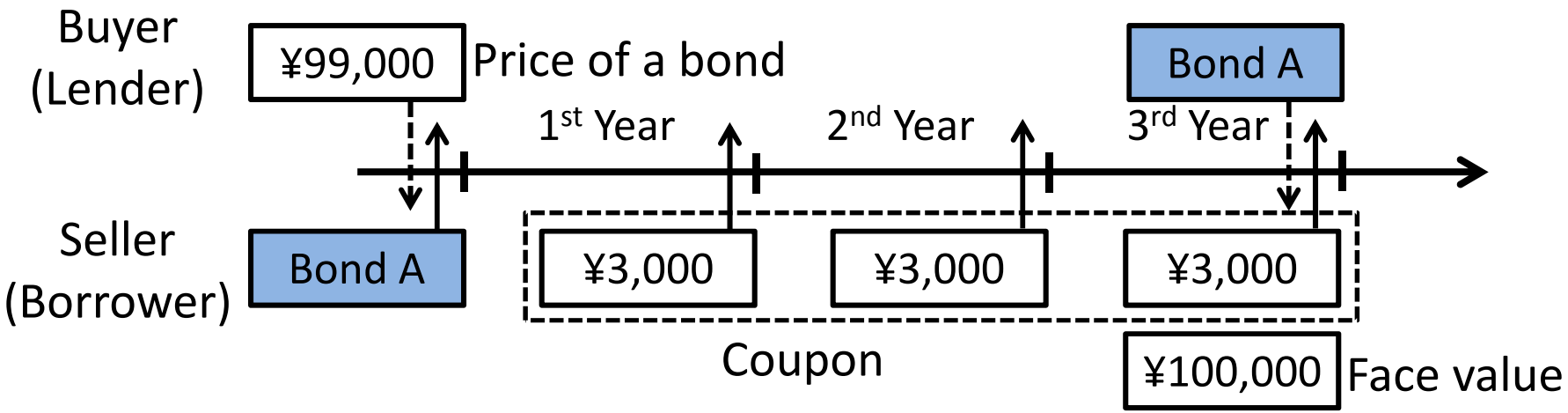


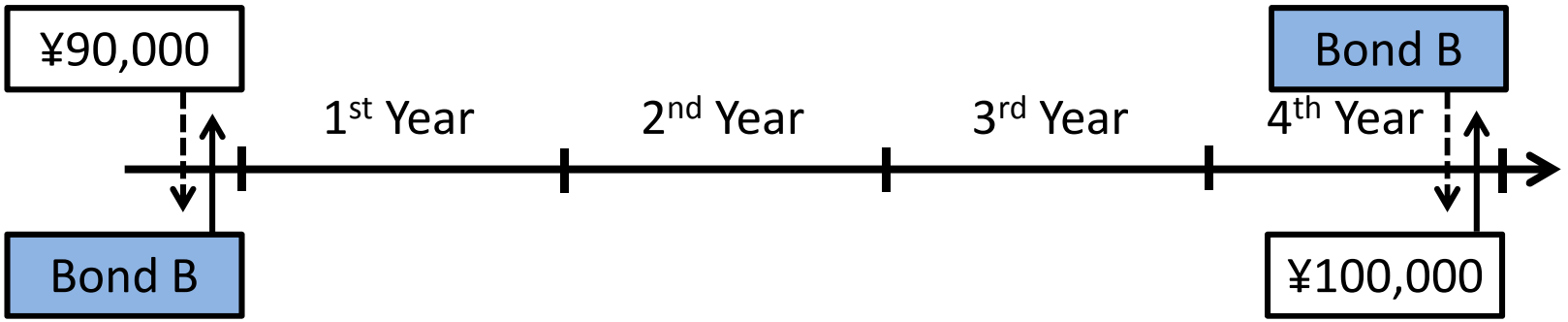
At an interest rate of 4 %, people hold exactly as much money as they want, thus have no incentive to take action.

→ Interest rates stays at 4%.

But what if the interest rate is 6% or 2%?
Do market push it toward 4%?

What is bond?





Calculating interest rates allows us to compare these two bonds directly.

Interest Rates

Interest rate: How much one unit of currency invested in an asset earns for one year

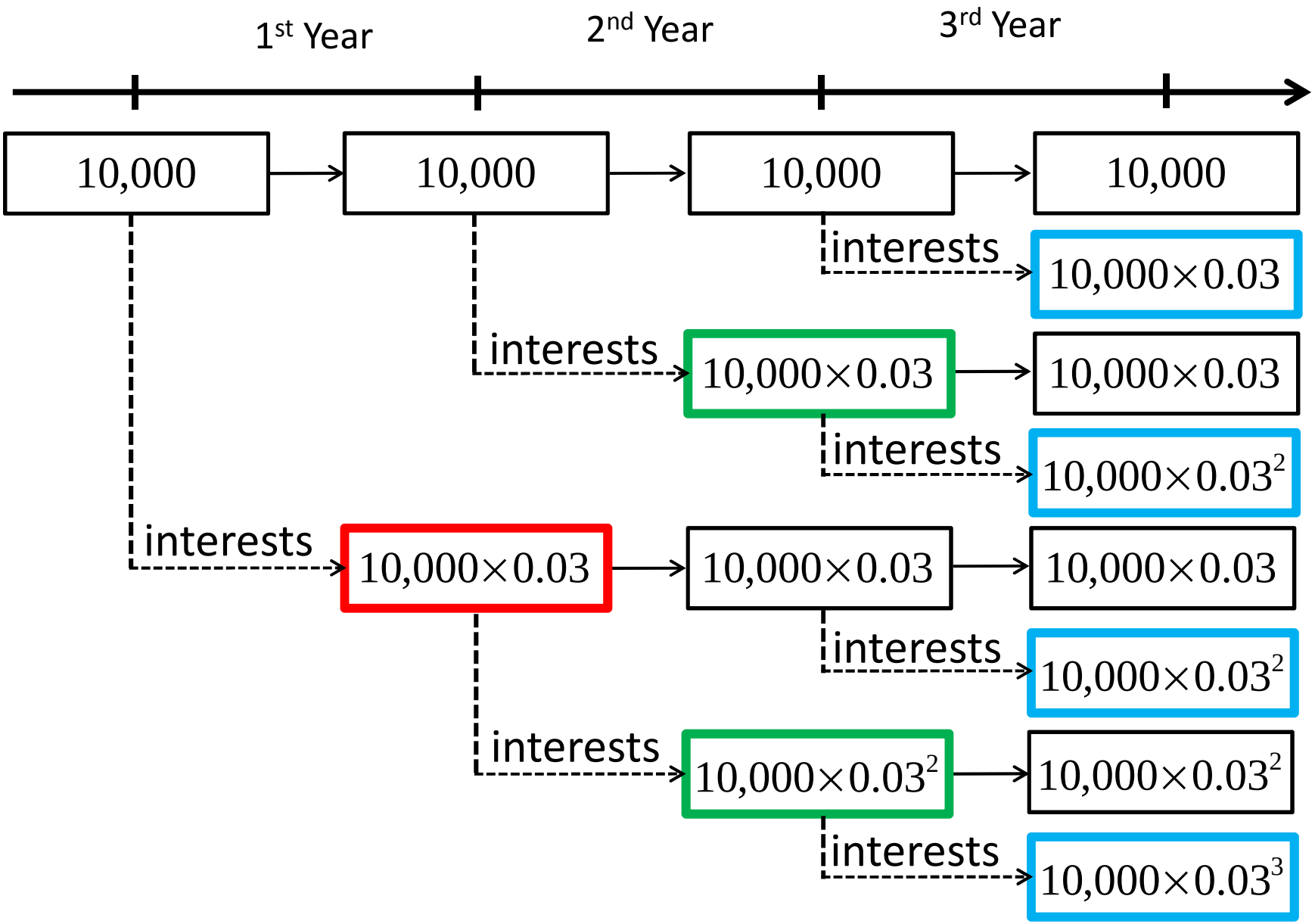
If you lend ¥10,000 with an interest rate of 0.03 for 3 years, the total amount you will receive at the maturity is;

$$10{,}000 \times \left(1+0.03\right)^3$$

If you lend P yens with an interest rate of i for n years, your money will be worth;

$$P \times \left(1+i\right)^n$$

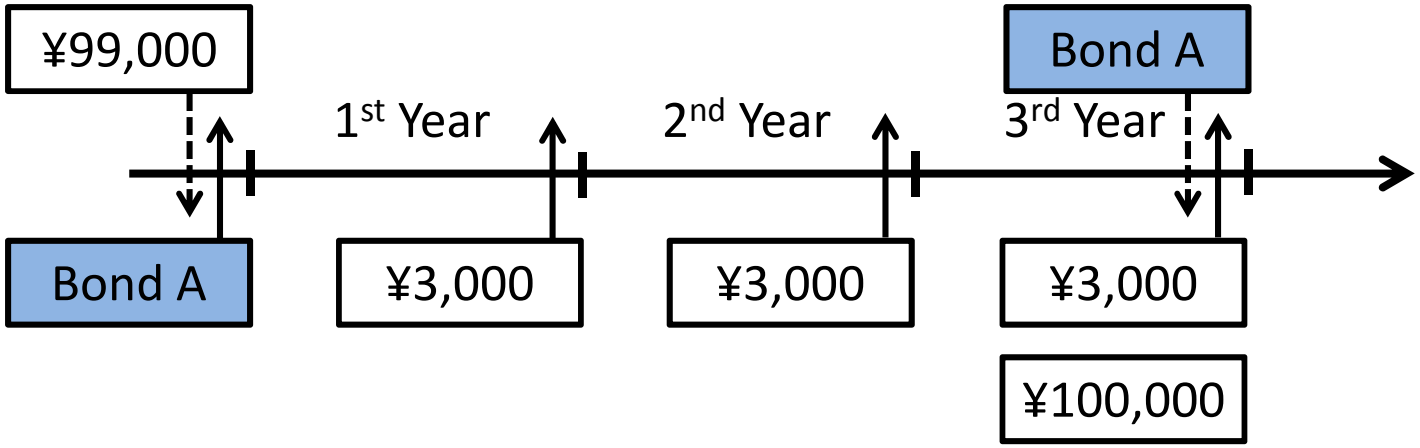
It can be verified in the following manner.


1st Year
2nd Year
3rd Year
10,000
10,000
10,000
10,000
interests
10,000×0.03
interests
10,000×0.03
10,000×0.03
interests
10,000×0.03²
interests
10,000×0.03
10,000×0.03
10,000×0.03
interests
10,000×0.03²
interests
10,000×0.03²
10,000×0.03²
interests
10,000×0.03³

Total payment at the maturity

$$
\begin{aligned}
&= 10{,}000 + (10{,}000 \times 0.03 + 10{,}000 \times 0.03 + 10{,}000 \times 0.03) \\
&\quad + (10{,}000 \times 0.03^2 + 10{,}000 \times 0.03^2 + 10{,}000 \times 0.03^2) \\
&\quad + 10{,}000 \times 0.03^3 \\
&= 10{,}000 + 10{,}000 \times 3 \times 0.03 + 10{,}000 \times 3 \times 0.03^2 \\
&\quad + 10{,}000 \times 0.03^2 \\
&= 10{,}000 \times (1 + 3 \times 0.03 + 3 \times 0.03^2 + 0.03^3) \\
&= 10{,}000 \times (1^3 + 3 \times 1^2 \times 0.03 + 3 \times 1 \times 0.03^2 + 0.03^3) \\
&= \boxed{10{,}000 \times (1 + 0.03)^3}
\end{aligned}
$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$


¥99,000
Bond A
1st Year
2nd Year
3rd Year
Bond A
¥3,000
¥3,000
¥3,000
¥100,000

How can we find the interest rate?

Divide ¥99,000 into three parts.

	Today	1 year after	2 years after	3 years after
A	a_1 →	3,000		
B	a_2 →		3,000	
C	a_3 →			3,000+100,000

A: a_1 is lent for one year and earns interests for one year

B: a_2 is lent for two years and earns interests for two years

C: a_3 is lent for three years and earns interests for three years

i … interest rate (unknown for the present)

Today	1 year after	2 years after	3 years after
A a_1	$a_1\times(1+i)=3{,}000$		
B a_2		$a_2\times(1+i)^2=3{,}000$	
C a_3			$a_3\times(1+i)^3=3{,}000+100{,}000$

$$a_1\times(1+i)=3{,}000 \longrightarrow a_1=\frac{3{,}000}{1+i}$$

$$a_2\times(1+i)^2=3{,}000 \longrightarrow a_2=\frac{3{,}000}{(1+i)^2}$$

$$a_3\times(1+i)^3=3{,}000+100{,}000 \longrightarrow a_3=\frac{3{,}000+100{,}000}{(1+i)^3}$$

Today	1 year after	2 years after	3 years after
A $\frac{3,000}{1+i}$ →	3,000		
B $\frac{3,000}{(1+i)^2}$ →		3,000	
C $\frac{3,000+100,000}{(1+i)^3}$ →			3,000+100,000

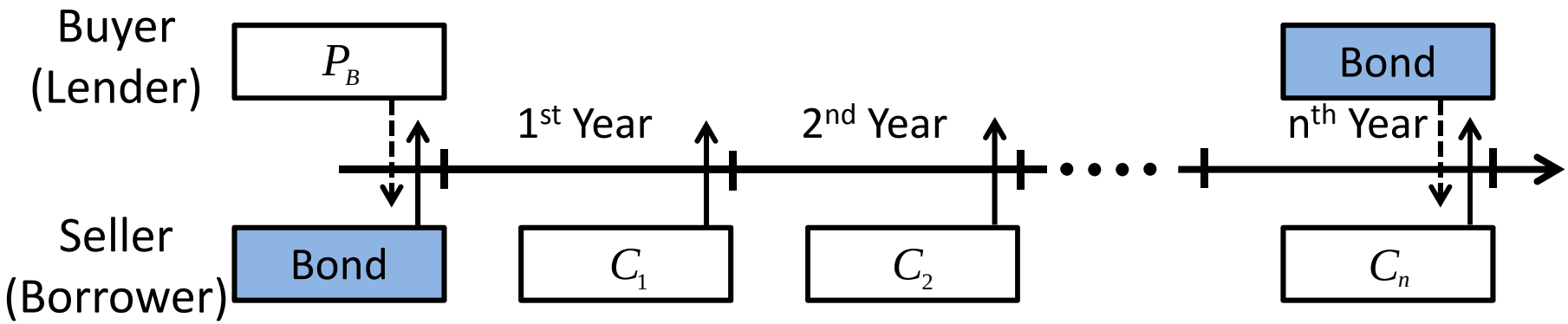
The sum of A, B and C must be equal to ¥99,000.

$$\frac{3,000}{1+i} + \frac{3,000}{(1+i)^2} + \frac{3,000+100,000}{(1+i)^3} = 99,000$$

This is an equation for the interest rate, and the solution gives the interest rate of Bond A.

$$i \approx 0.0356$$

General Case

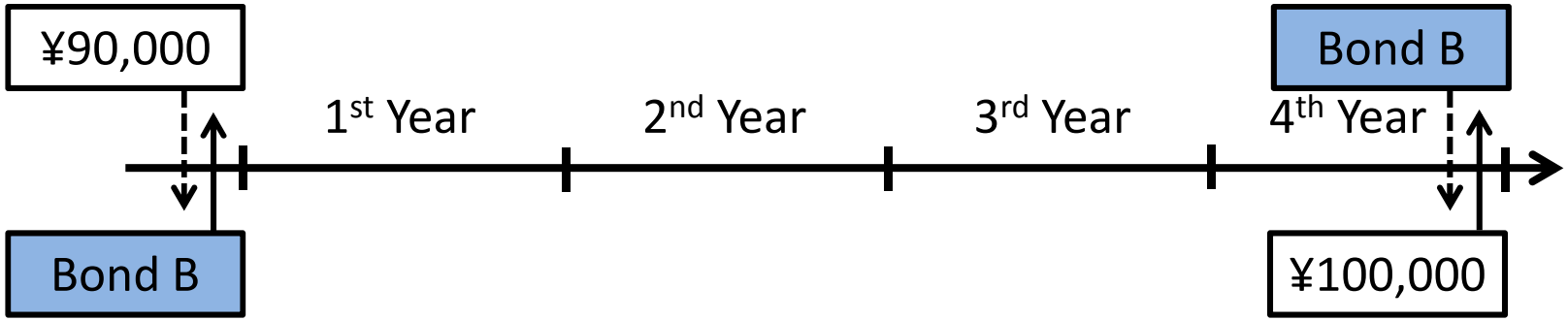


$$\frac{C_1}{1+i} + \frac{C_2}{(1+i)^2} + \cdots + \frac{C_n}{(1+i)^n} = P_B$$

If the stream of payments and the price of a bond are given, the equation gives the interest rate that the bond offers.

The equation also shows how the price of a bond and its interest rate are related.

Interest Rates: Example



$$n=4$$
$$C_1=C_2=C_3=0$$
$$P_B=90{,}000$$
$$C_4=100{,}000$$

$$\frac{0}{1+i}+\frac{0}{(1+i)^2}+\frac{0}{(1+i)^3}+\frac{100{,}000}{(1+i)^4}=90{,}000$$

$$i\approx 0.0267$$

Bond Price and Interest Rate: Example

What if you buy Bond A at a lower price?

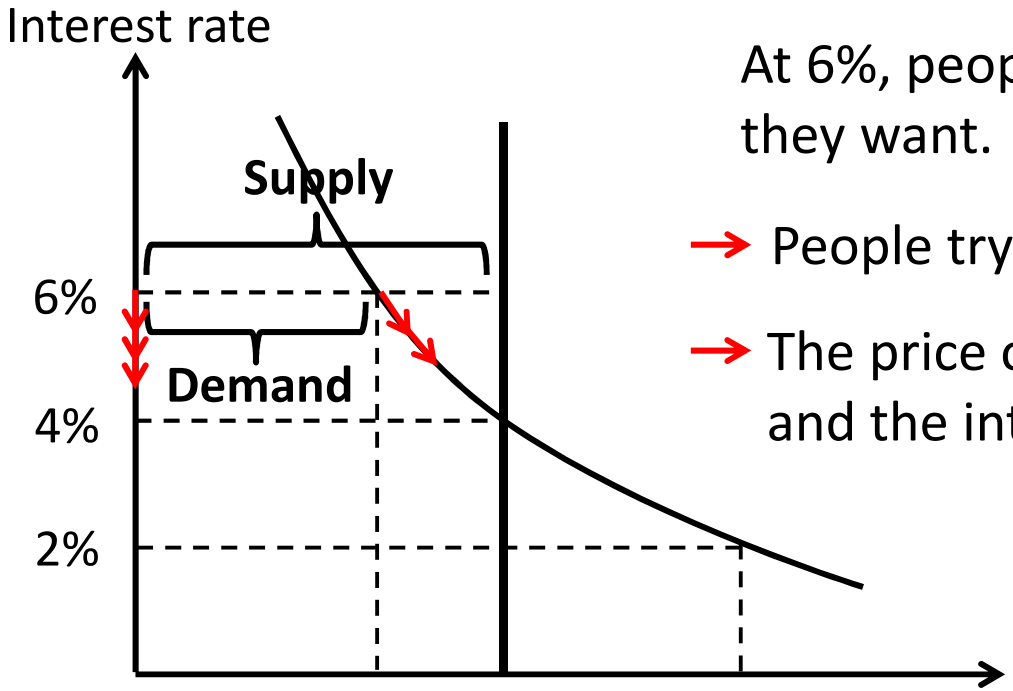
$$\frac{3{,}000}{1+i}+\frac{3{,}000}{(1+i)^2}+\frac{3{,}000+100{,}000}{(1+i)^3}=\boxed{99{,}000} \qquad i=0.0356$$

$$\downarrow \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \downarrow$$

$$\frac{3{,}000}{1+i}+\frac{3{,}000}{(1+i)^2}+\frac{3{,}000+100{,}000}{(1+i)^3}=\boxed{95{,}000} \qquad i=0.0483$$

With the stream of payments constant, as the price of a bond falls, its interest rate rises.

Equilibrium Interest Rate (1)



At 6%, people hold more money than they want.

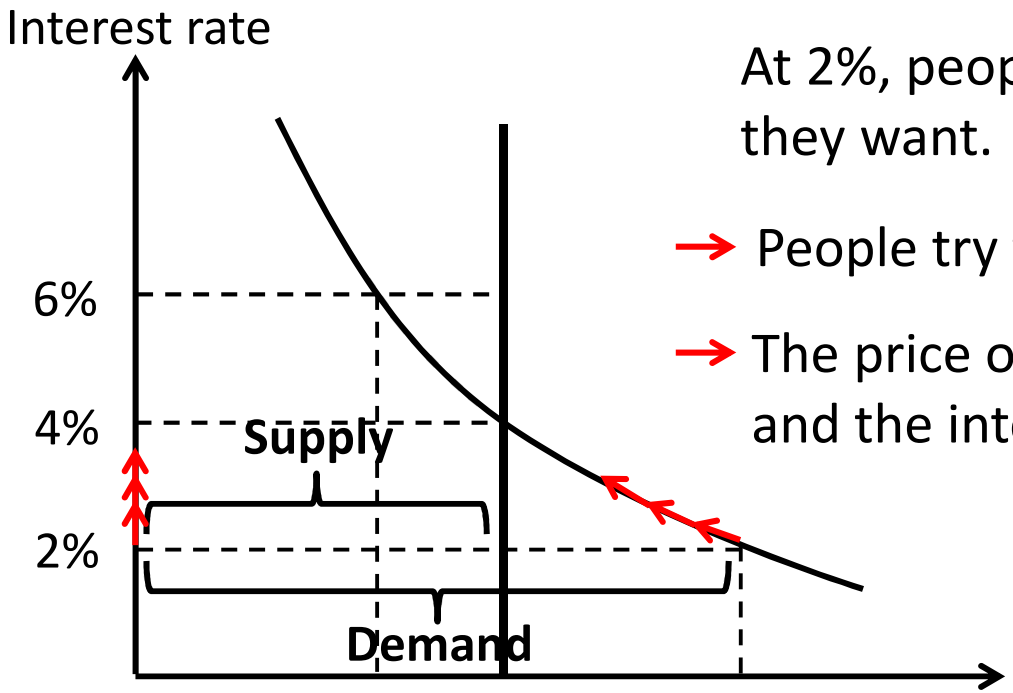
→ People try to exchange money for bonds.

→ The price of a bond rises, and the interest rate falls.

→ The cost of holding money falls.

→ The demand for money rises.

Equilibrium Interest Rate (2)



At 2%, people hold less money than they want.

→ People try to exchange bonds for money.

→ The price of a bond falls, and the interest rate rises.

→ The cost of holding money rises.

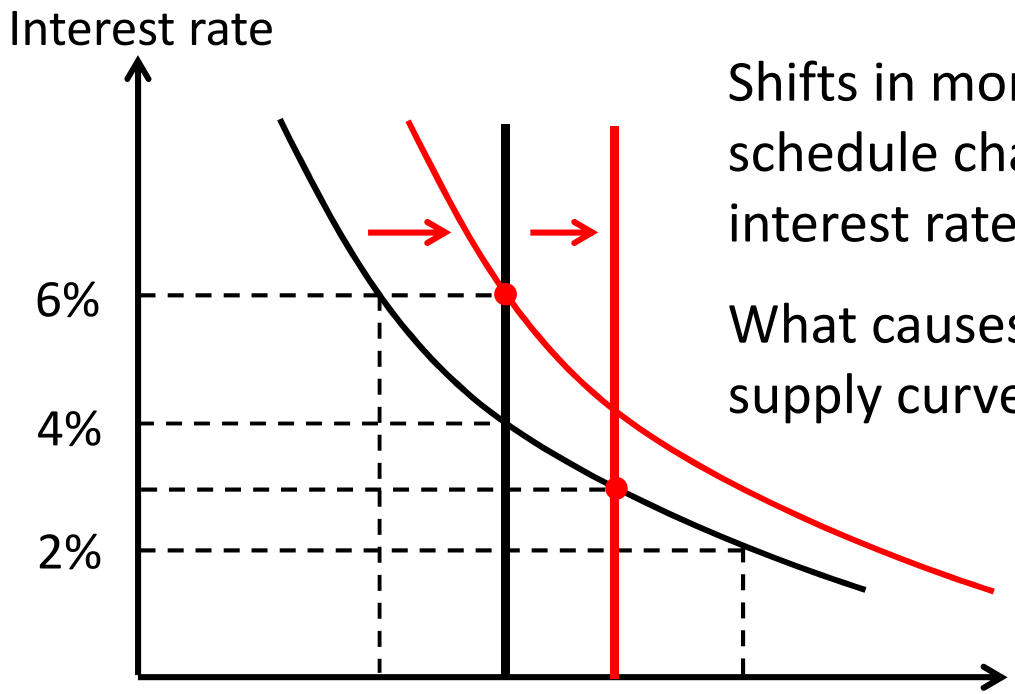
→ The demand for money falls.

Equilibrium Interest Rate (3)

- At interest rates higher than 4%
  Because people hold more money than they want, they try to exchange money for bonds, raising the price of bonds and lowering the interest rate.
- At interest rates lower than 4%
  Because people hold less money than they want, they try to exchange bonds for money, lowering the price of bonds and raising the interest rate.
- At 4%
  Because people hold exactly as much money as they want, they do not take another action and the interest rate stays.

The interest rate of bonds is determined so that the demand for money could be just equal to the outstanding stock of money.

Changes in Equilibrium Interest Rates

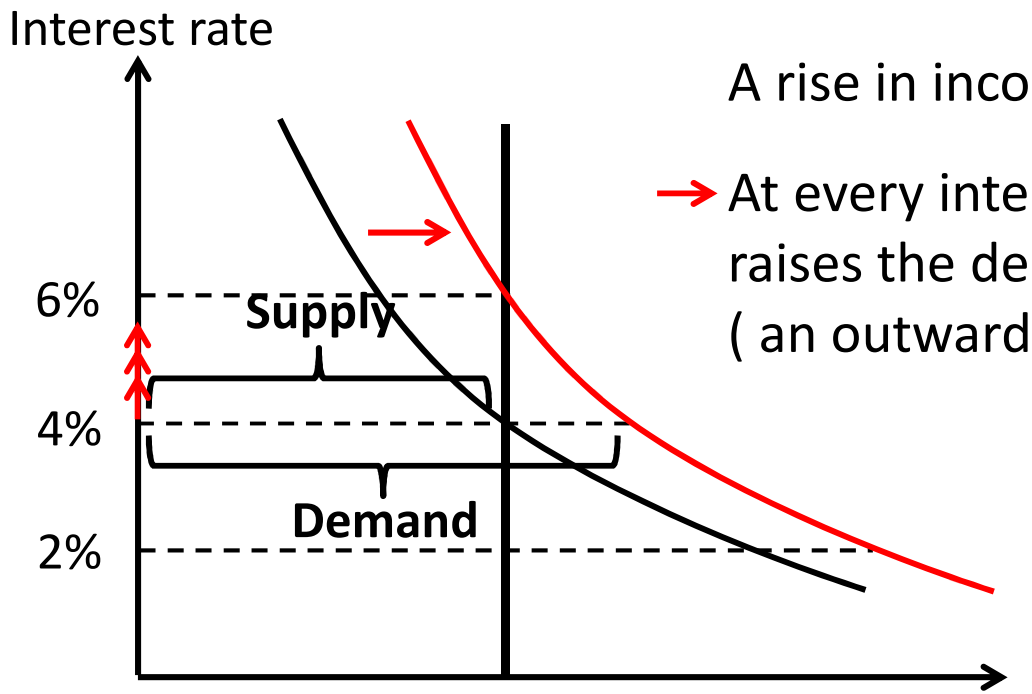


Shifts in money demand or/and supply schedule change the equilibrium interest rate.

What causes money demand or/and supply curves to shift?

- Changes in GDP shift the demand schedule.
- Changes in price level shift the demand schedule.
- Changes in money stock shift the supply schedule.

Changes in GDP(income)



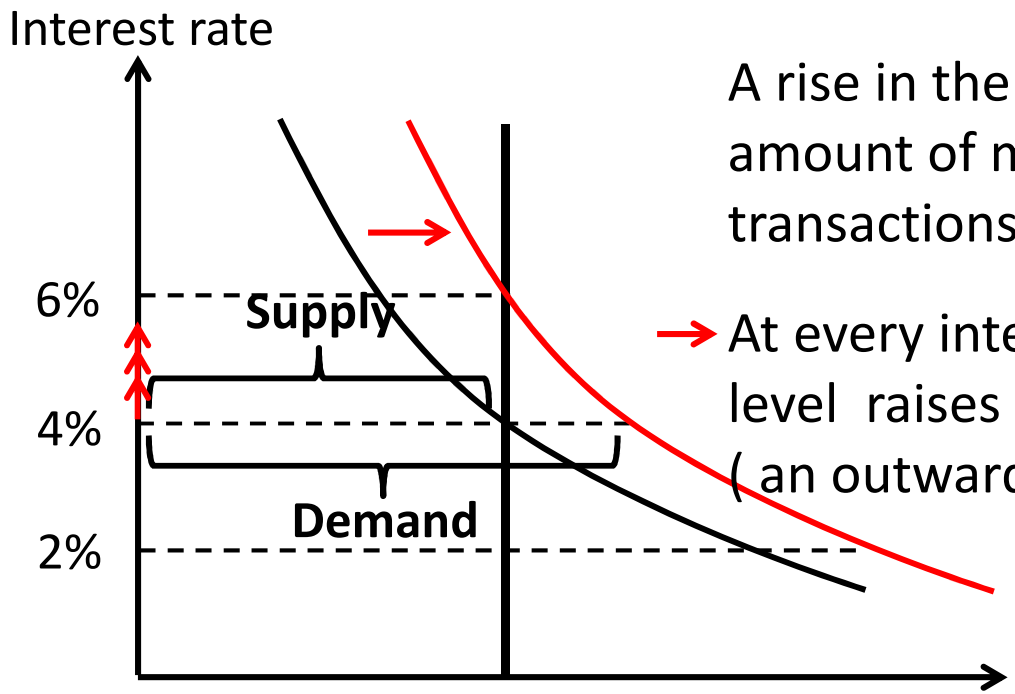
A rise in income increases transactions.

→ At every interest rate, a rise in income raises the demand for money.
(an outward shift in demand schedule)

→ At the initial equilibrium interest rate, people hold less money than they want.

→ A rise(fall) in income raises(lowers) the equilibrium interest rate.

Changes in the Price Level



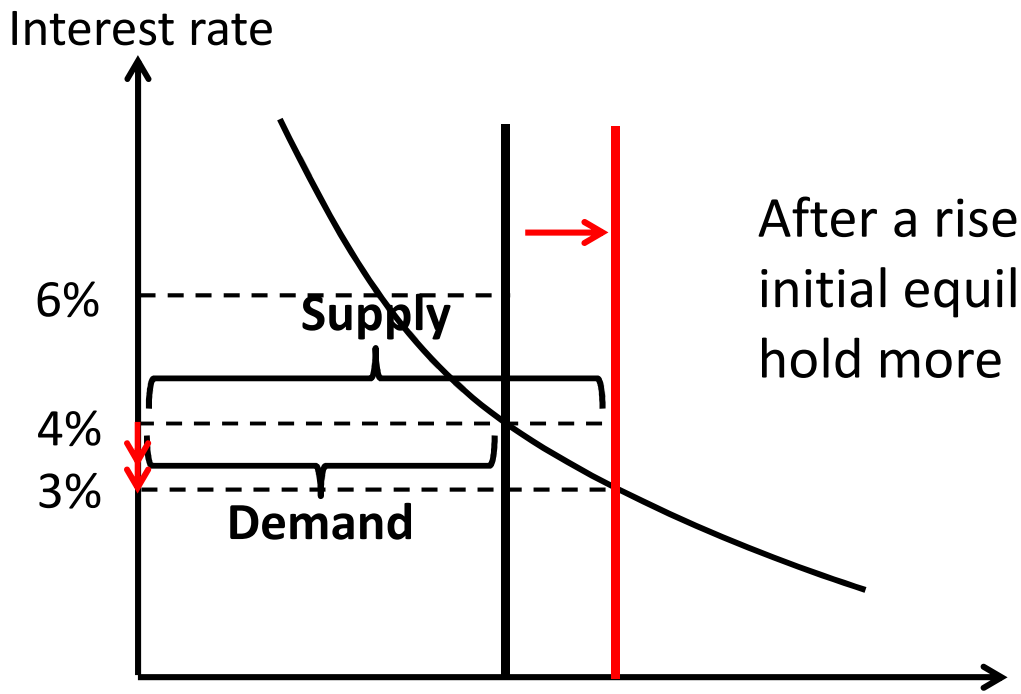
A rise in the price level increases the amount of money that we need for transactions.

→ At every interest rate, a rise in the price level raises the demand for money.
(an outward shift in demand schedule)

→ At the initial equilibrium interest rate, people hold less money than they want.

→ A rise(fall) in price level raises(lowers) the equilibrium interest rate.

Changes in the Money Stock



After a rise in the money stock, at the initial equilibrium interest rate, people hold more money than they want.

→ People try to buy bonds for their money.

→ A rise(fall) in the money stock lowers(raises) the equilibrium interest rate.

Events/Shocks	Effects on the equilibrium interest rate
Rise in GDP	Rise in interest rate
Fall in GDP	Fall in interest rate
Rise in price level	Rise in interest rate
Fall in price level	Fall in interest rate
Rise in money stock	Fall in interest rate
Fall in money stock	Rise in interest rate

Liquidity Preference Model