# Fiscal & Financial System in Japan A

2010 Spring

## Session 10

## The Term Structure of Interest Rates(2)

July 5, 2010

Hideyuki IWAMURA

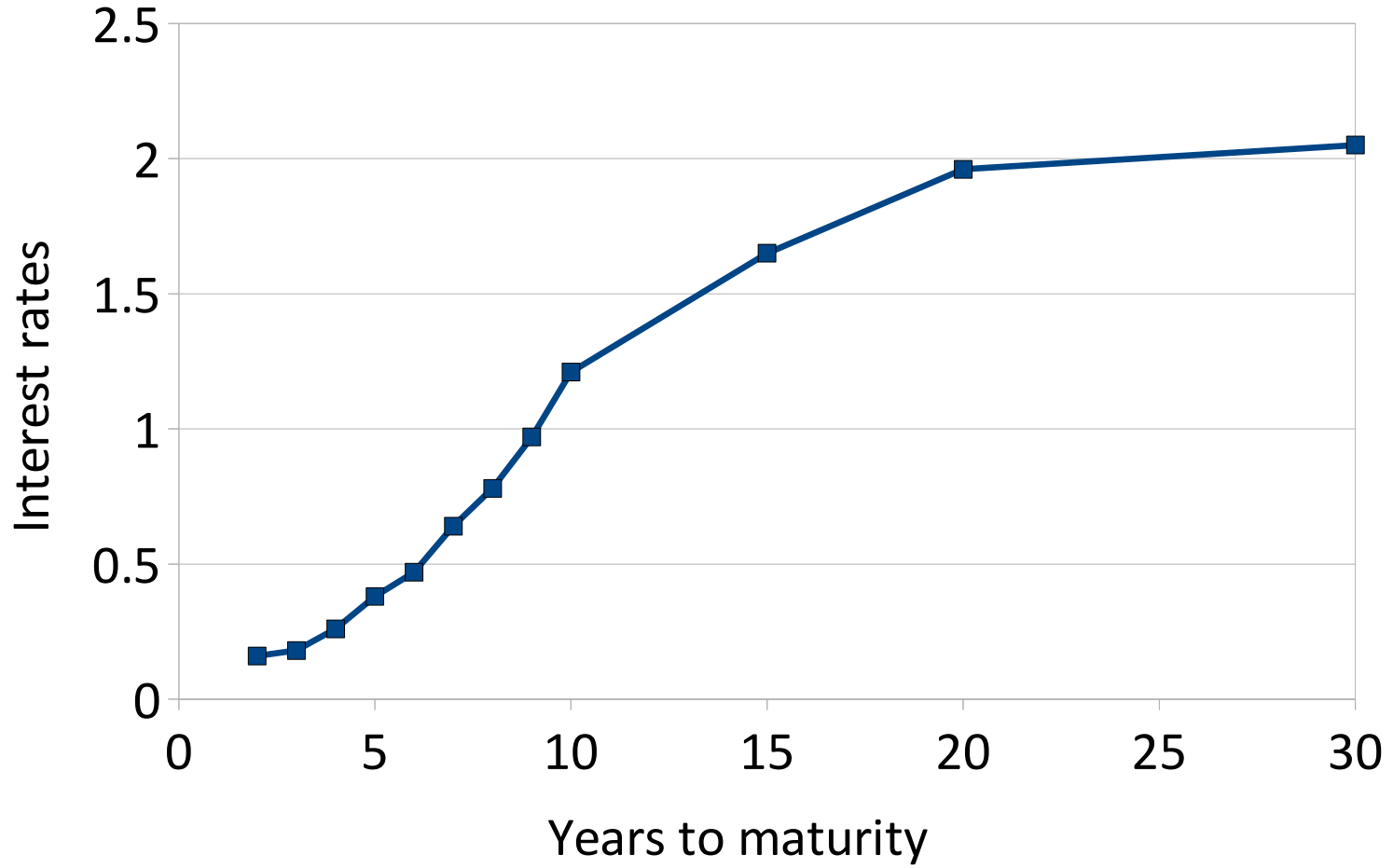
Senior Lecturer

Faculty of International Studies

MEIJI GAKUIN UNIVERSITY

# Yield Curve

Yield Curve … Plot of yields on bonds with different maturities, but the same risk and liquidity




Example: Yield curve for Japanese government bonds

# Three Empirical Facts on Yield Curves

Fact 1 — Interest rates on bonds with different maturities move together over time.

Fact 2 — When short-term interest rates are low, yield curves tend to slope upward;
when short-term interest rates are high, yield curves tend to slope downward.

Fact 3 — Yield curves almost always slope upward.

Economists have suggested *expectations theory*, to explain these facts we actually observe.

# Expectations Theory

*Theory* or *Model* is a set of assumptions on the behavior of people.

**Assumption 1. Perfect substitutes**

For investors, bonds with different maturities are *perfect substitutes*.

Investors do not prefer one maturity to another, if the other conditions are equal.

**Assumption 2. Risk-neutrality**

Investors are *risk-neutral*.

Investors care only about the *expected* yields, whatever the variability is.

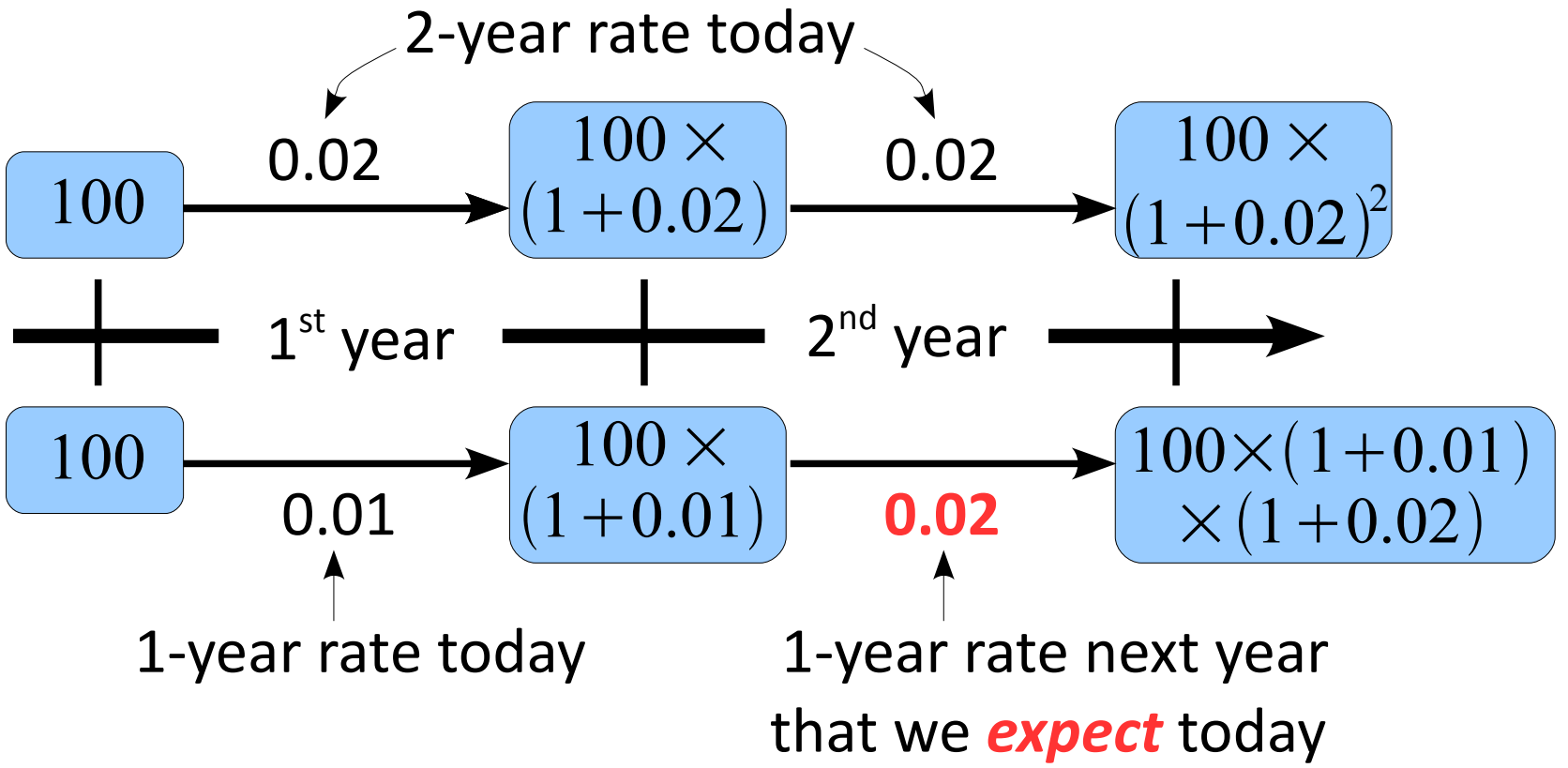
# Implication of the Theory

Suppose that

(1) the yield on 2-year bond today is 0.02,

(2) the yield on 1-year bond today is 0.01, and

(3) the yield on 1-year bond next year is *expected to be* 0.02



Perfect substitutability and risk-neutrality implies...

If yield on buy-and-hold > (expected) yield on roll-over

→ Everyone tries to replace 1-year bond with 2-year bond.

→ Rise in 2-year bond price, fall in 1-year bond price today

→ Fall in 2-year yield, rise in 1-year yield today

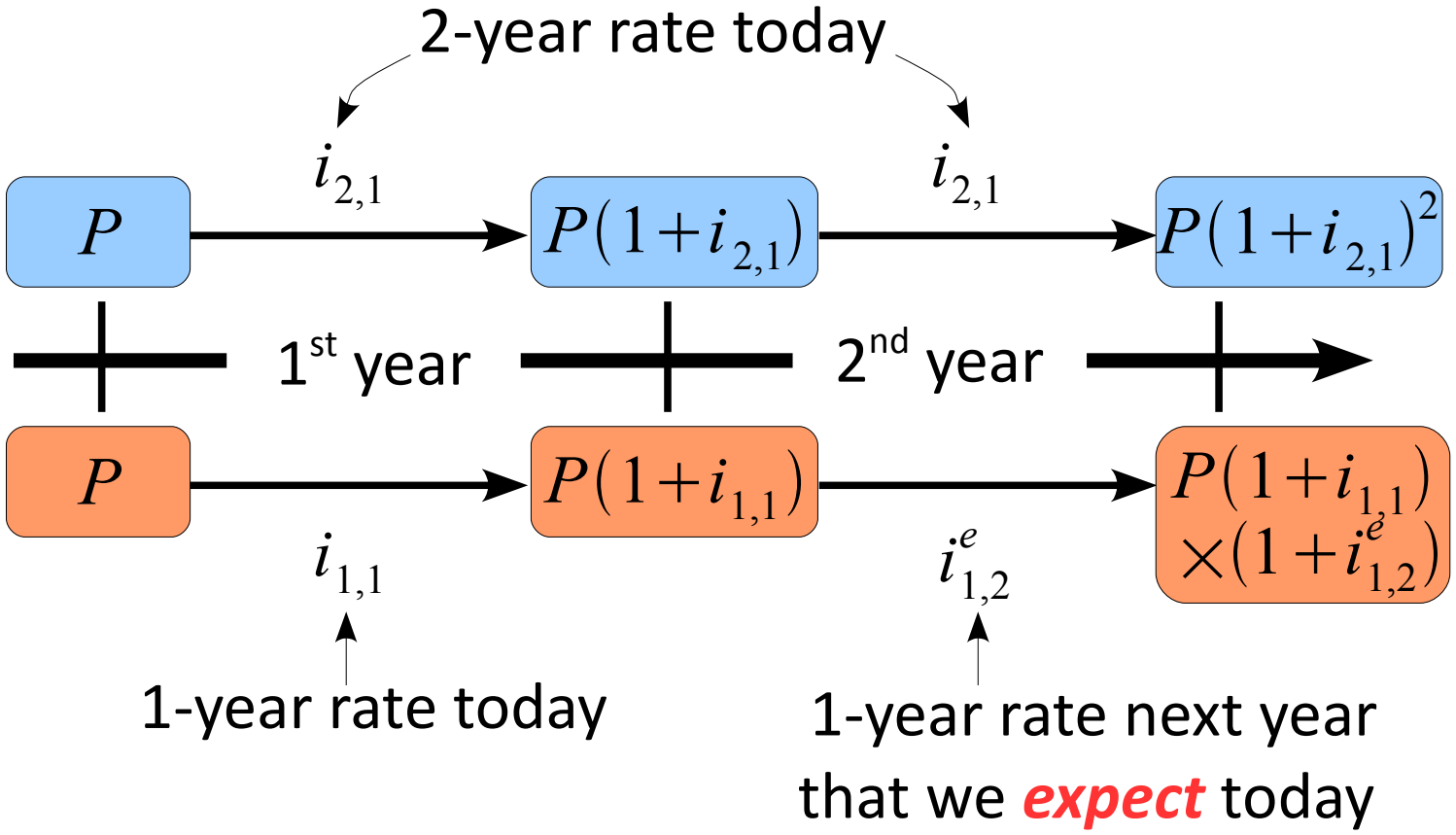
The process continues until

yield on buy-and-hold = (expected) yield on roll-over

This specifies the relationship among the yields on 1-year and 2-year bonds.

**The term structure of interest rates**

# Notation


2-year rate today
$i_{2,1}$
$i_{2,1}$
$P$
$P(1+i_{2,1})$
$P(1+i_{2,1})^2$
1st year
2nd year
$P$
$P(1+i_{1,1})$
$P(1+i_{1,1})\times(1+i^e_{1,2})$
$i_{1,1}$
$i^e_{1,2}$
1-year rate today
1-year rate next year
that we *expect* today

Buy-and-hold Roll-over

$$P(1+i_{2,1})^2 = P(1+i_{1,1})(1+i^e_{1,2})$$

$$1+2i_{2,1}+(i_{2,1})^2 = 1+i_{1,1}+i^e_{1,2}+i_{1,1}i^e_{1,2}$$

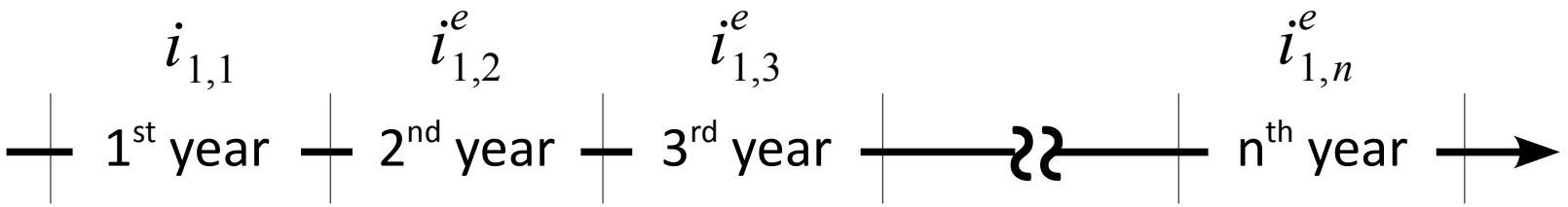
Very small and negligible

$$2i_{2,1} = i_{1,1}+i^e_{1,2}$$

$$i_{2,1} = \frac{i_{1,1}+i^e_{1,2}}{2}$$

The yield on 2-year bond today equals

the average of
- (1) the yield on 1-year bond today
- (2) the *expected* yield on 1-year bond next year




$$i_{n,1} = \frac{i_{1,1} + i^e_{1,2} + i^e_{1,3} + \cdots + i^e_{1,n}}{n}$$

The yield on n-year bond today equals

the average of
- (1) the yield on 1-year bond today
- (2) the *expected* yield on 1-year bond a year ahead
- (3) the *expected* yield on 1-year bond two years ahead
- ⋮
- (n) the *expected* yield on 1-year bond n-1 years ahead

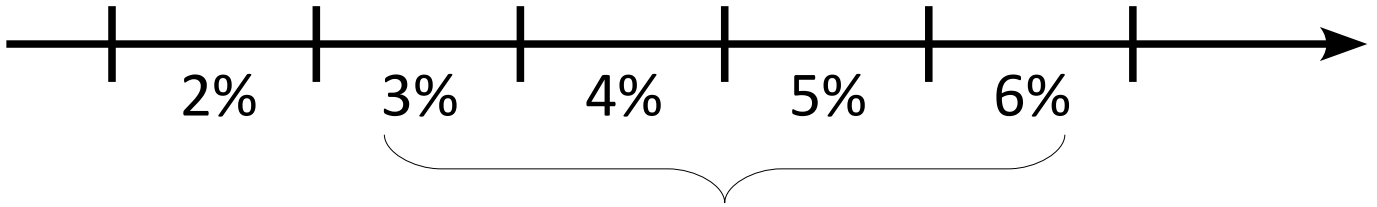
The theory implies that

the yields on longer maturities depend on people's **expectations** of 1-year yields in the future.

**Expectations Theory**

# Numerical Examples

Example 1 People expect short-term rates to rise in the future




Today's interest rate on 2-year bond $\frac{2+3}{2} = 2.5\%$

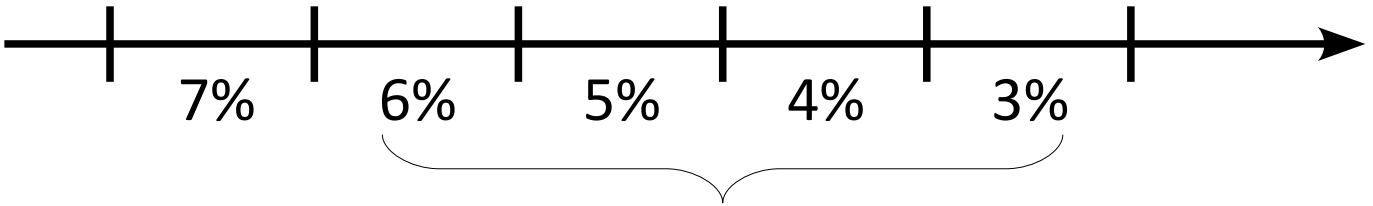
Today's interest rate on 3-year bond $\frac{2+3+4}{3} = 3\%$

Today's interest rate on 4-year bond $\frac{2+3+4+5}{4} = 3.5\%$

Today's interest rate on 5-year bond $\frac{2+3+4+5+6}{5} = 4\%$

**Upward-sloping**
yield curve

Example 2 People expect short-term rates to fall in the future




Today's interest rate on 2-year bond $\frac{7+6}{2} = 6.5\%$

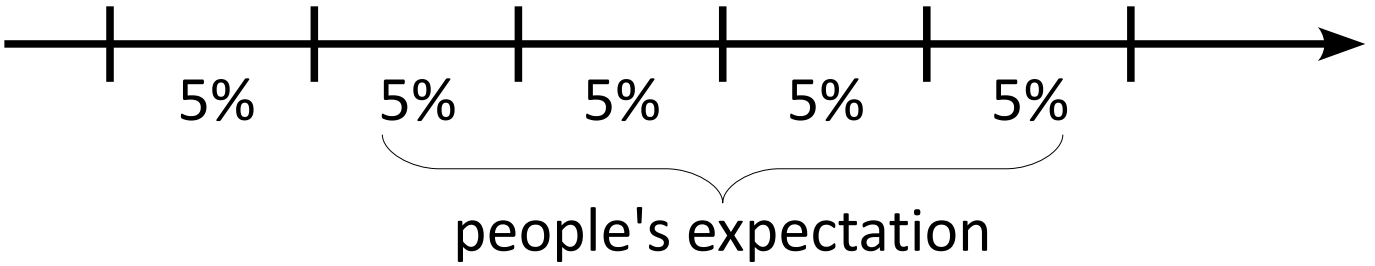
Today's interest rate on 3-year bond $\frac{7+6+5}{3} = 6\%$

Today's interest rate on 4-year bond $\frac{7+6+5+4}{4} = 5.5\%$

Today's interest rate on 5-year bond $\frac{7+6+5+4+3}{5} = 5\%$

**Downward-sloping**
yield curve

Example 3 People expect short-term rates to stay in the future




Today's interest rate on 2-year bond $\frac{5+5}{2} = 5\%$

Today's interest rate on 3-year bond $\frac{5+5+5}{3} = 5\%$

Today's interest rate on 4-year bond $\frac{5+5+5+5}{4} = 5\%$

Today's interest rate on 5-year bond $\frac{5+5+5+5+5}{5} = 5\%$

**Flat** yield curve

# Expectation ⇔ Yield Curves

When people expect short-term rates to { rise / stay / fall } in the future,

the yield curve slopes { upward / flat / downward }

{ Upward- / Flat- / Downward- } sloping yield curve implies that

people expect short-term rates to { rise / stay / fall } in the future.

# Expectations Theory and Fact 2

When a short-term rate today is relatively low ...

- ➔ It is more likely to rise (or return to some *normal level* ).
- ➔ People expect short-term rates to rise in the future.
- ➔ Expectations theory predicts upward-sloping yield curve.

When a short-term rate today is relatively high ...

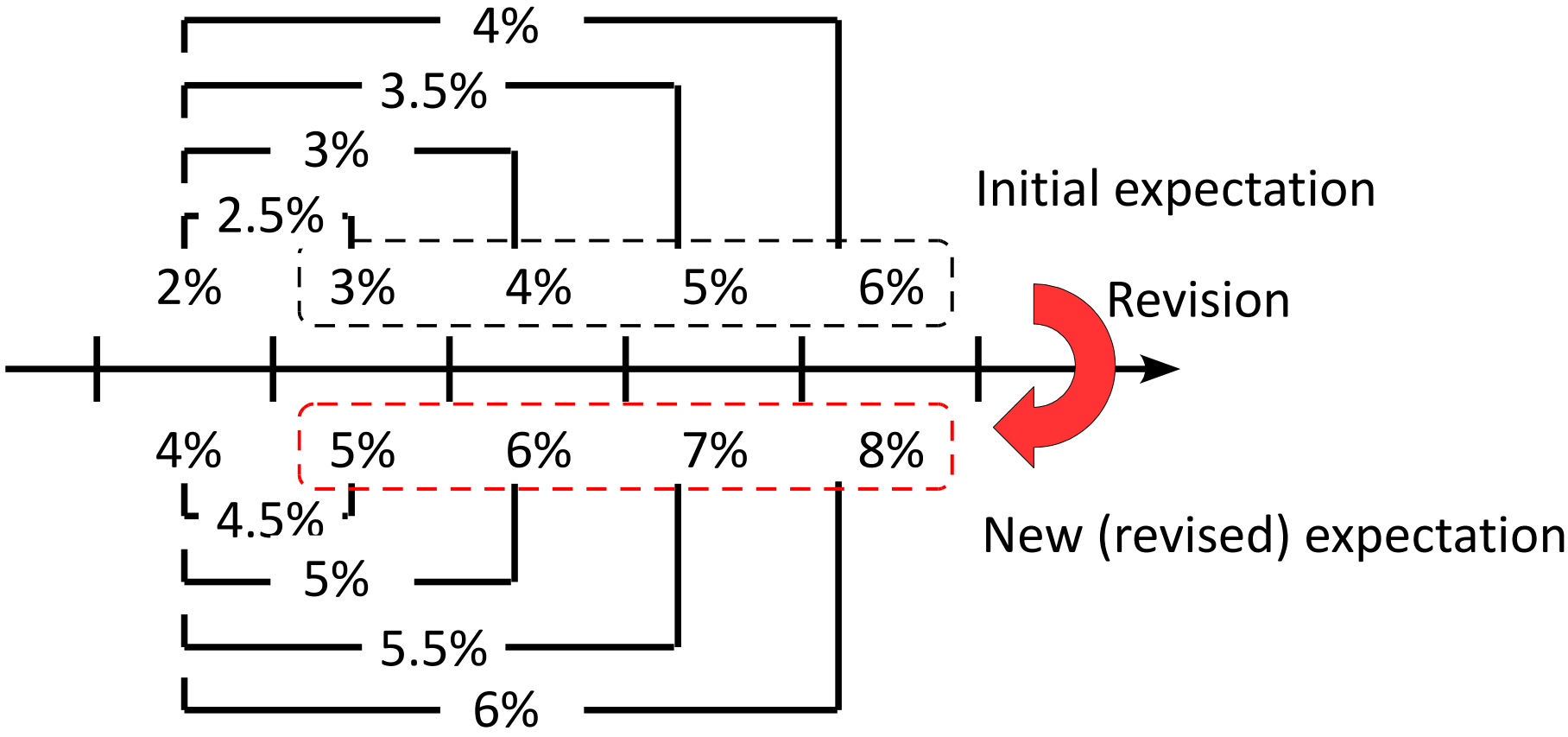
- ➔ It is more likely to fall (or return to some *normal level* ).
- ➔ People expect short-term rates to fall in the future.
- ➔ Expectations theory predicts downward-sloping yield curve.

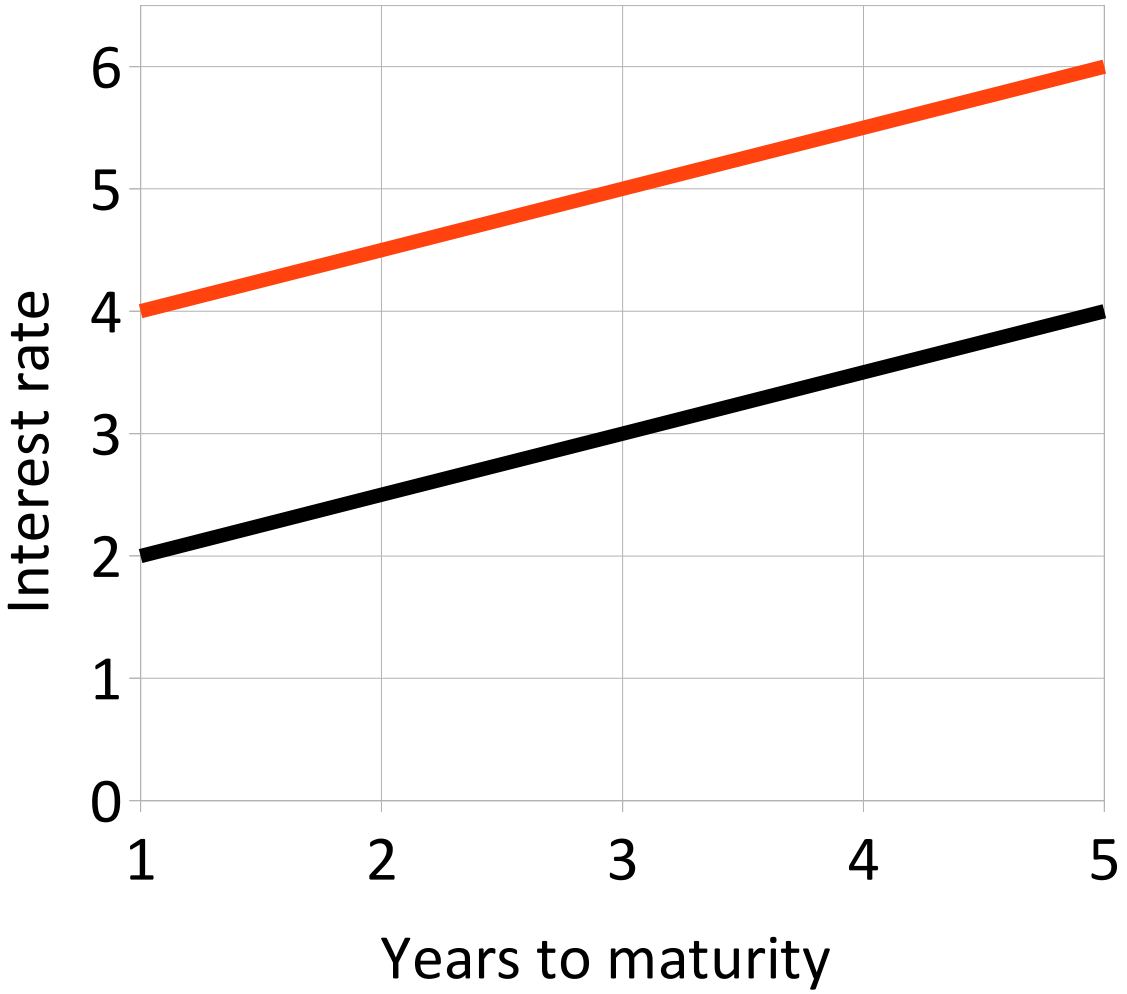
Predictions by expectations theory are consistent with “Fact 2.”

# Expectations Theory and Fact 1

Suppose that the central bank raises a short-term rate today.

People interpret the action as a change in monetary policy and revise their expectations on the future short-term rates upward.






Predictions by expectations theory are consistent with “Fact 1.”