Fiscal & Financial System in Japan A

2010 Spring

Session 5 Interest Rates (continued)

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Four Types of Financial Instruments

Suppose you have 100,000 yen to invest.

Four different offers have been made by four entrepreneurs.

(1) Simple loan with a 0.05 interest rate and a maturity of 3 years

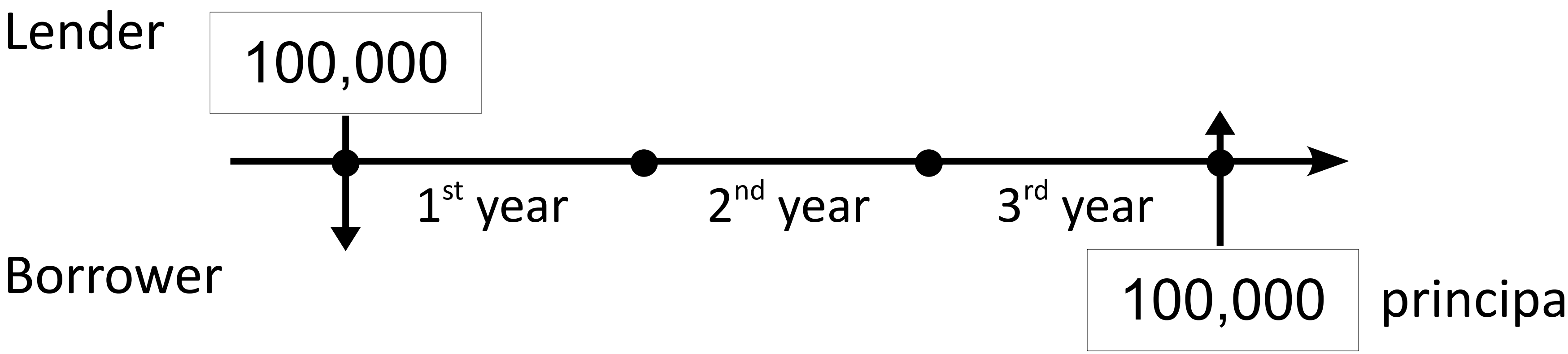
(2) Fixed-payment loan with a maturity of 4 years
and a 29,000 yen repayment each year

(3) Discount bond with a face value of 102,000 yen
and a maturity of 1 year

(4) Coupon bond with a face value of 100,000 yen,
a coupon rate of 0.05, and a maturity of 4 years

1 Simple Loan

A simple loan of 100,000 yen with an interest rate of 5 % and a maturity of 3 years



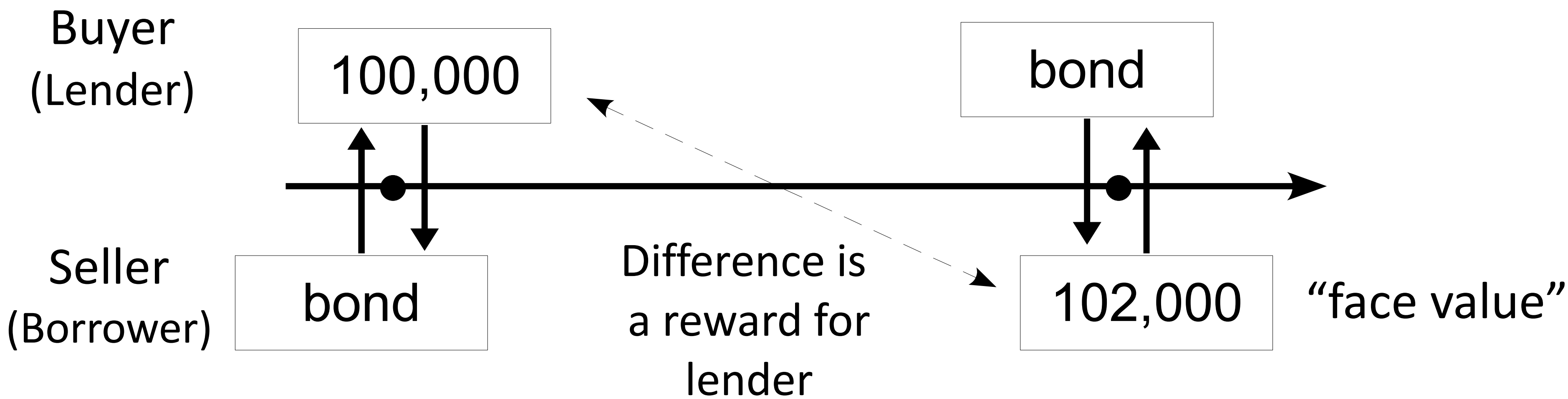
principal + interests = $100{,}000 \times (1+0.05)^3$

15,762.5 interest

principal + interests = principal × $(1+i)^n$

2 Discount Bond

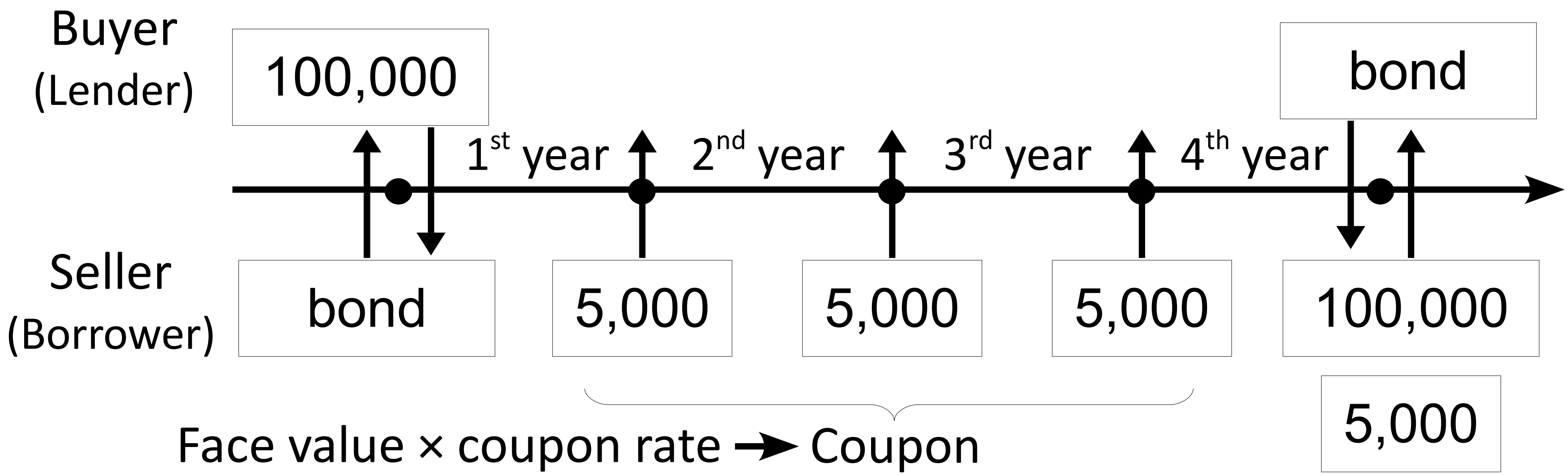
If you buy a one-year discount bond sold at 100,000 yen
with a face value of 102,000 yen …



The issuer sells a bond at 100,000 yen
which he promises to repurchase at its face value after one year.

Coupon Bond

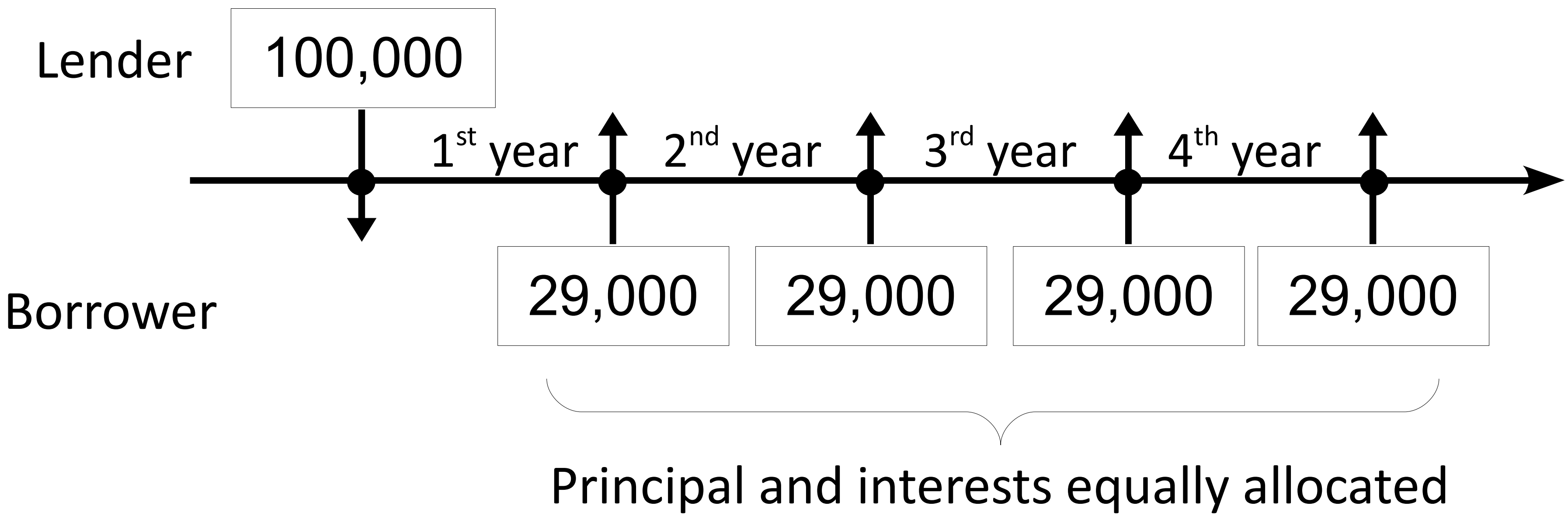
If you buy a coupon bond with a face value of 100,000 yen, a coupon rate of 0.05, and a maturity of 4 years ...



The issuer sells a bond at its face value which pays to the holder a "coupon" each period and, at the maturity, he buys back at the face value.

Fixed-Payment Loan

If you make your friend a fixed-payment loan of 100,000 yen with a maturity of 4 years and <u>annual payments</u> of 29,000 yen...

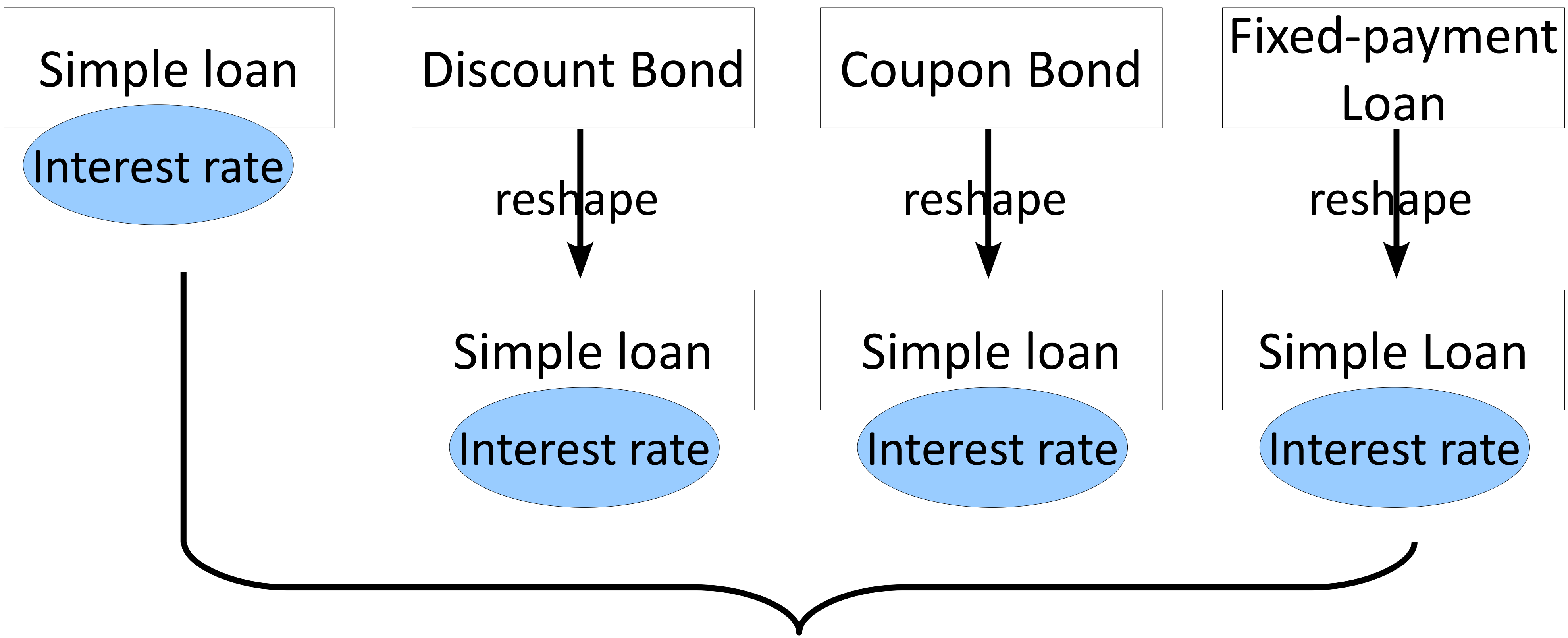


Borrower repays the <u>same fixed</u> amount of money at every period which consists of the principal and interests.

<u>No lump-sum payment</u> at the maturity.

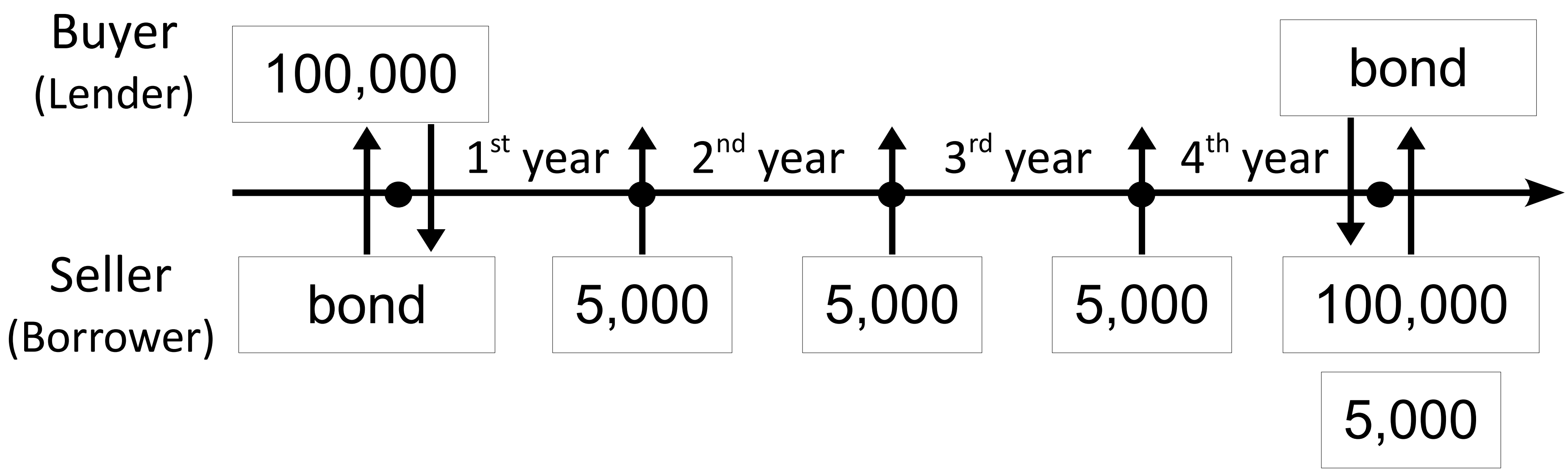
How do we compare?

Reshape the instruments into simple loans and compare their interest rates.



Determine which implicitly offers the greatest interest rate!

Reshaping a Coupon Bond: example

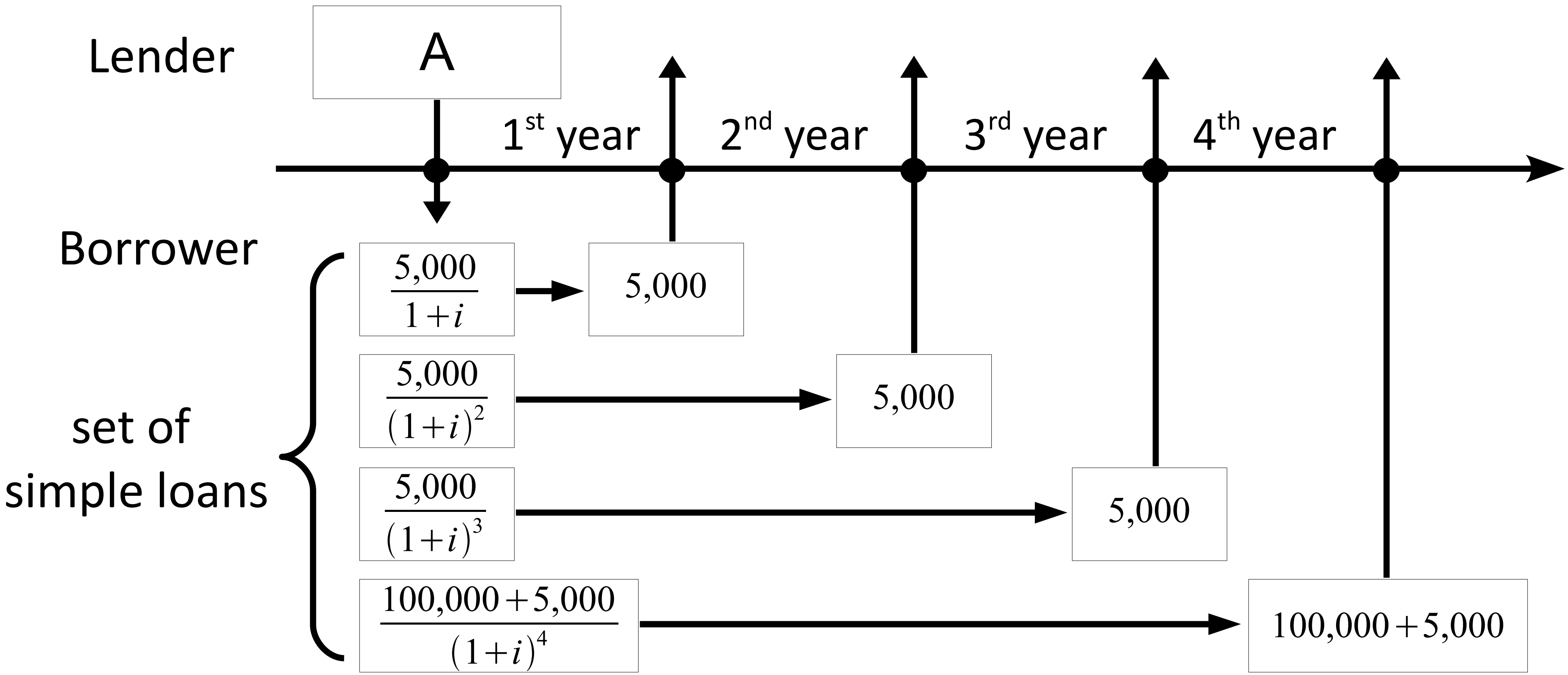


How can we reshape this coupon bond into a simple loan that gives the same income at the same timing?

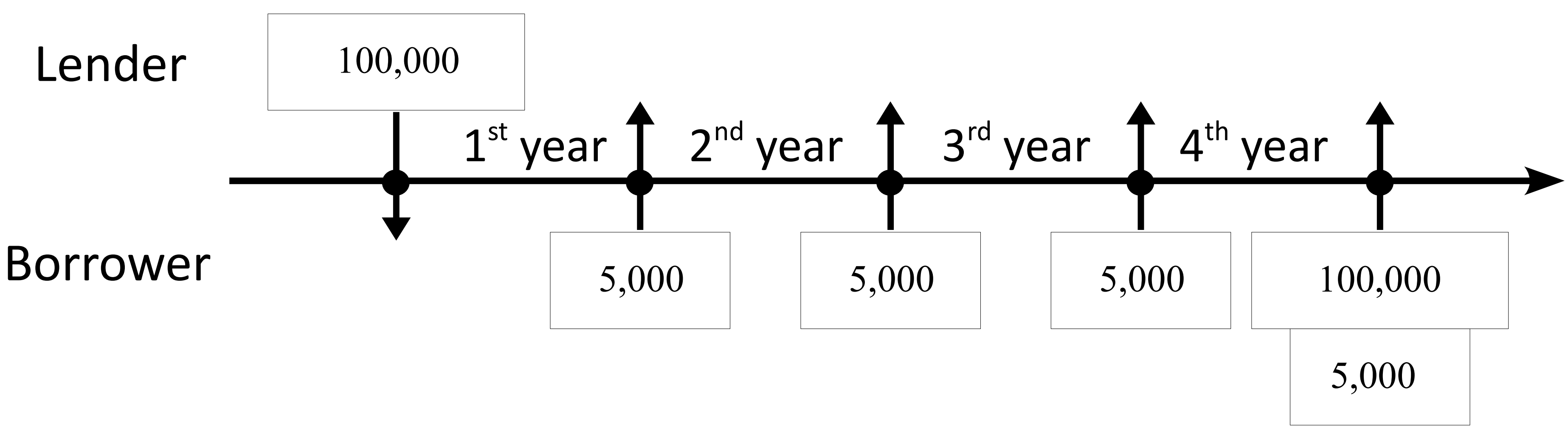
Interest rate = i

	1st year	2nd year	3rd year	4th year
$\frac{5{,}000}{1+i}$	$5{,}000$			
$\frac{5{,}000}{(1+i)^2}$	$\frac{5{,}000}{1+i}$	$5{,}000$		
$\frac{5{,}000}{(1+i)^3}$	$\frac{5{,}000}{(1+i)^2}$	$\frac{5{,}000}{1+i}$	$5{,}000$	
$\frac{5{,}000}{(1+i)^4}$	$\frac{5{,}000}{(1+i)^3}$	$\frac{5{,}000}{(1+i)^2}$	$\frac{5{,}000}{1+i}$	$5{,}000$
$\frac{100{,}000}{(1+i)^4}$	$\frac{100{,}000}{(1+i)^3}$	$\frac{100{,}000}{(1+i)^2}$	$\frac{100{,}000}{1+i}$	$100{,}000$

The amount of money we initially need,
in order to receive the same timeline of incomes
from a simple loan with an interest rate of i



Lender
A
1st year
2nd year
3rd year
4th year
Borrower
set of simple loans
$\frac{5,000}{1+i}$
5,000
$\frac{5,000}{(1+i)^2}$
5,000
$\frac{5,000}{(1+i)^3}$
5,000
$\frac{100,000+5,000}{(1+i)^4}$
100,000+5,000



Lender
100,000
1st year
2nd year
3rd year
4th year
Borrower
5,000
5,000
5,000
100,000
5,000

Yield to Maturity

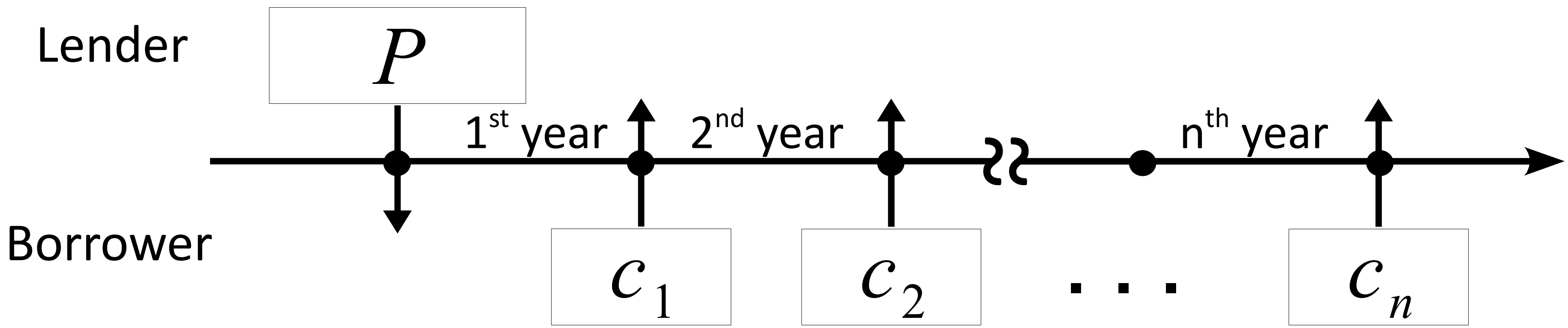
Initial payment for a coupon bond = Initial payment for a simple loan that offers the same timeline of cashflows

$$100{,}000 = \frac{5{,}000}{1+i} + \frac{5{,}000}{(1+i)^2} + \frac{5{,}000}{(1+i)^3} + \frac{5{,}000+100{,}000}{(1+i)^4}$$

With the interest rate that satisfies this equation,
a simple loan of the same 100,000 yen gives the same income at the same timing as our coupon bond.

→ **YIELD TO MATURITY or simply INTEREST RATE**

Calculation of Interest Rates



$$P = \frac{c_1}{1+i} + \frac{c_2}{(1+i)^2} + \cdots + \frac{c_n}{(1+i)^n}$$

P , n , and $c_{1,} c_{2,.} \cdot\cdot , c_n$ given,
this equation gives the yield to maturity (interest rate) of the financial instrument.

Example 1

(1) <u>Simple loan</u> with a 0.05 interest rate and a maturity of 3 years

(2) <u>Fixed-payment loan</u> with a maturity of 4 years and a 29,000 yen repayment each year

$$100{,}000 = \frac{29{,}000}{1+i} + \frac{29{,}000}{(1+i)^2} + \frac{29{,}000}{(1+i)^3} + \frac{29{,}000}{(1+i)^4}$$

(3) <u>Discount bond</u> with a face value of 102,000 yen and a maturity of 1 year

$$100{,}000 = \frac{102{,}000}{1+i}$$

(4) <u>Coupon bond</u> with a face value of 100,000 yen, a coupon rate of 0.05, and a maturity of 4 years

$$100{,}000 = \frac{5{,}000}{1+i} + \frac{5{,}000}{(1+i)^2} + \frac{5{,}000}{(1+i)^3} + \frac{5{,}000 + 100{,}000}{(1+i)^4}$$

Examples 2

5-year discount bond with a face value of 100,000 yen sold at 90,000 yen

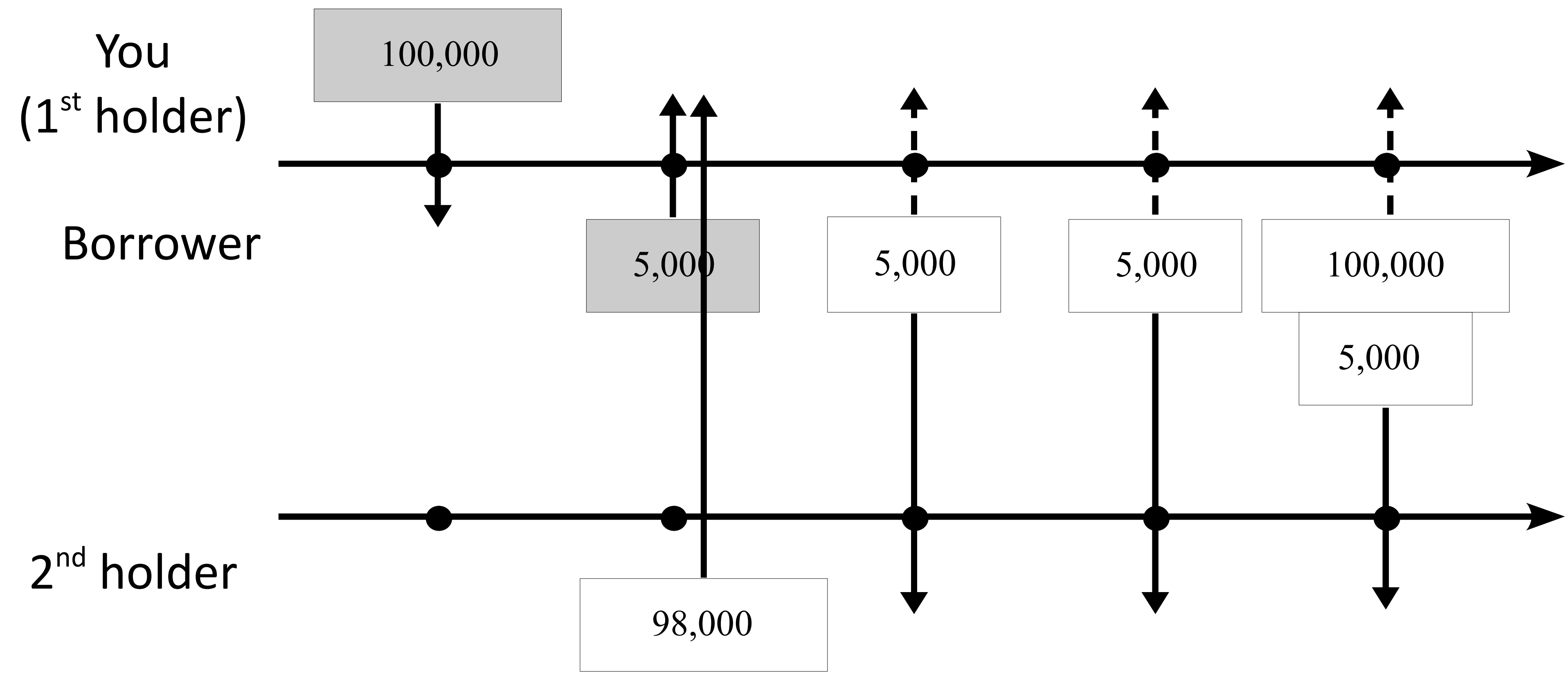
$$90{,}000 = \frac{0}{1+i} + \frac{0}{(1+i)^2} + \frac{0}{(1+i)^3} + \frac{0}{(1+i)^4} + \frac{100{,}000}{(1+i)^5}$$

20-year commercial mortgage of 10,000,000 yen
with annual payments of 1,273,100 yen

$$10{,}000{,}000 = \frac{1{,}273{,}100}{1+i} + \frac{1{,}273{,}100}{(1+i)^2} + \cdots + \frac{1{,}273{,}100}{(1+i)^{20}}$$

Bond Prices and Interest Rates

If you sell the bond in a secondary market ...



What is the interest rate for the new holder?

The interest rate for the 2nd holder is given by ...

$$98{,}000 = \frac{5{,}000}{1+i} + \frac{5{,}000}{(1+i)^2} + \frac{5{,}000+100{,}000}{(1+i)^3}$$

$\Rightarrow i = 0.0574$

What if he could buy at lower P ?

$$P = \frac{5{,}000}{1+i} + \frac{5{,}000}{(1+i)^2} + \frac{5{,}000+100{,}000}{(1+i)^3}$$

Fall in bond price $P\downarrow$ $\longleftrightarrow$ Rise in interest rate $i\uparrow$

Rise in bond price $P\uparrow$ $\longleftrightarrow$ Fall in interest rate $i\downarrow$

Bond prices and interest rates are **NEGATIVELY** related.

Numerical Example

$$P = \frac{5{,}000}{1+i} + \frac{5{,}000}{(1+i)^2} + \frac{5{,}000 + 100{,}000}{(1+i)^3}$$

How does the interest rate change with the bond price?

Bond prices	Interest rates
100,000 (face value)	0.05
98,000	0.0574
96,000	0.0651
94,000	0.0730
92,000	0.0811
90,000	0.0895

Calculation with MS-EXCEL

$$98{,}000 = \frac{5{,}000}{1+i} + \frac{5{,}000}{(1+i)^2} + \frac{5{,}000+100{,}000}{(1+i)^3}$$

Such a non-linear equation can not easily be solved.

Find an approximate value of i that nearly satisfies the equation.

Substitute an arbitrary value, for example, $i = 0.05$.

If LHS < RHS, then substitute a larger value, say, $i = 0.06$.

Repeat the process until LHS and RHS are close enough and you get an approximate value of i.

Iterated calculation is easily done with the use of MS EXCEL's “Goal Seek.”